3.8 – EXERCÍCIO – pg. 79

1 - Seja $f(x)=\begin{cases} x-1 & , x\leq 3 \\ 3x-7 & , x>3 \end{cases}$

Calcule:

(a) $\lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^-}(x-1)=3-1=2$

(b) $\lim\limits_{x\to 3^+} f(x)=\lim\limits_{x\to 3^+}(3x-7)=3\cdot 3-7=2$

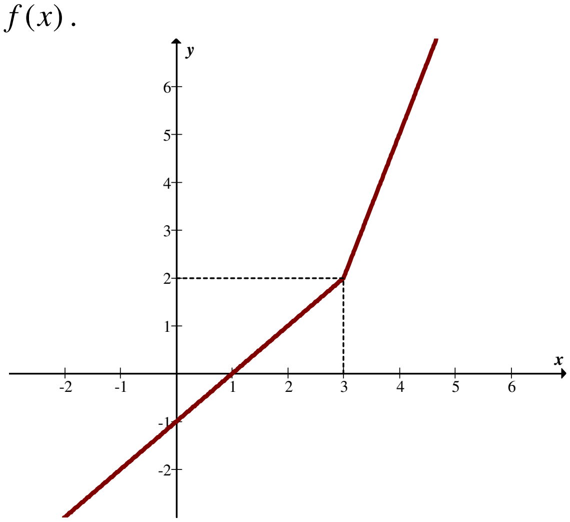
(c) $\lim\limits_{x\to 3} f(x)=2$

(d) $\lim\limits_{x\to 5^-} f(x)=3\cdot 5-7=8$

(e) $\lim\limits_{x\to 5^+} f(x)=8$

(f) $\lim\limits_{x\to 5} f(x)=8$

Esboçar o gráfico de $f(x)$.

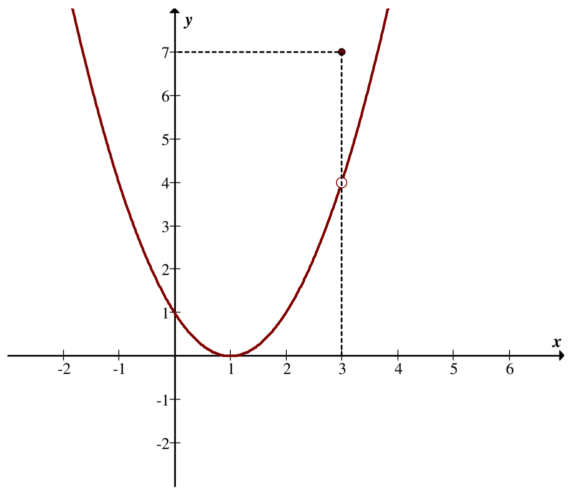


2 – Seja $h(x)=\begin{cases} x^2-2x+1 & , x\neq 3 \\ 7 & , x=3 \end{cases}$

Calcule $\lim_{x\to 3} h(x)$. Esboce o gráfico de *h(x).*

$$\left.\begin{array}{l}\lim_{x\to 3^-} h(x)=4\\ \lim_{x\to 3^+} h(x)=4\end{array}\right\}\Rightarrow \lim_{x\to 3} h(x)=4$$

Segue o gráfico



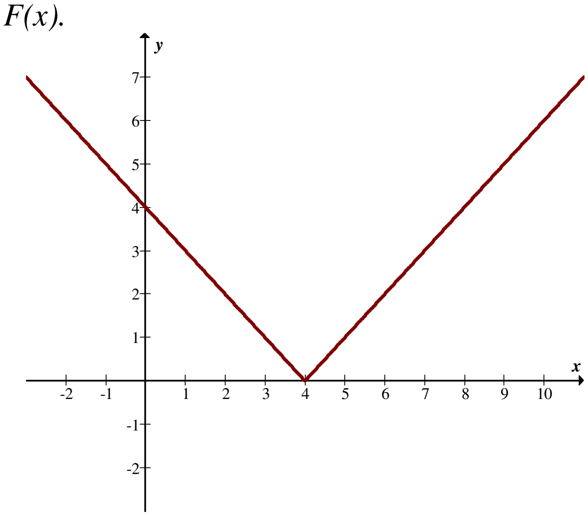
3 – Seja $F(x)=|x-4|$. Calcule os limites indicados se existirem:

(a) $\lim_{x\to 4^+} F(x)=\lim_{x\to 4^+}(x-4)=0$

(b) $\lim_{x\to 4^-} F(x)=\lim_{x\to 4^-}(-x+4)=0$

(c) $\lim_{x\to 4} F(x)=0$

Esboce o gráfico de *F(x).*



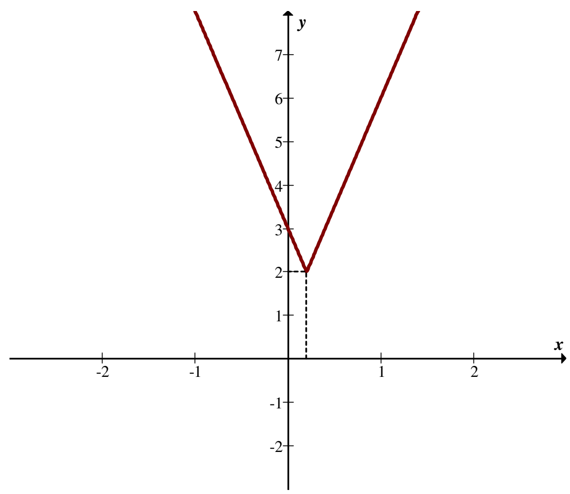
4 – Seja $f(x) = 2 + |5x - 1|$. Calcule se existir:

(a) $\lim\limits_{x \to \frac{1}{5}^+} f(x) = \lim\limits_{x \to \frac{1}{5}^+} \left[2 + (5x - 1)\right] = 2 + \cancel{5} \cdot \frac{1}{\cancel{5}} - 1 = 2$

(b) $\lim\limits_{x \to \frac{1}{5}^-} f(x) = \lim\limits_{x \to \frac{1}{5}^-} \left[2 - (5x - 1)\right] = 2 - (\cancel{5} \cdot \frac{1}{\cancel{5}} - 1) = 2 - 0 = 2$

(c) $\lim\limits_{x \to \frac{1}{5}} f(x) = 2$

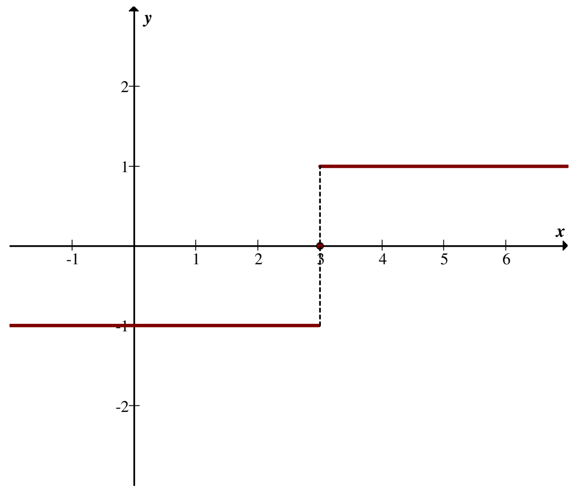
Esboce o gráfico de *f(x).*



5 - Seja $g(x) = \begin{cases} \dfrac{|x - 3|}{x - 3} & , x \neq 3 \\ 0 & , x = 3 \end{cases}$

(a) Esboce o gráfico de *g(x)*
(b) Achar $\lim\limits_{x \to 3^+} g(x)$, $\lim\limits_{x \to 3^-} g(x)$ e $\lim\limits_{x \to 3} g(x)$

(a) Segue o gráfico da função dada



(b) $\lim\limits_{x \to 3^+} g(x) = \lim\limits_{x \to 3^+} \dfrac{x-3}{x-3} = 1$; $\quad \lim\limits_{x \to 3^-} g(x) = \lim\limits_{x \to 3^-} \dfrac{-(x-3)}{x-3} = -1$ $\qquad \lim\limits_{x \to 3} g(x) \nexists$

6 – Seja $h(x) = \begin{cases} x/|x| & se\ x \neq 0 \\ 0 & se\ x = 0 \end{cases}$

Mostrar que *h(x)* não tem limite no ponto 0.

Temos que:

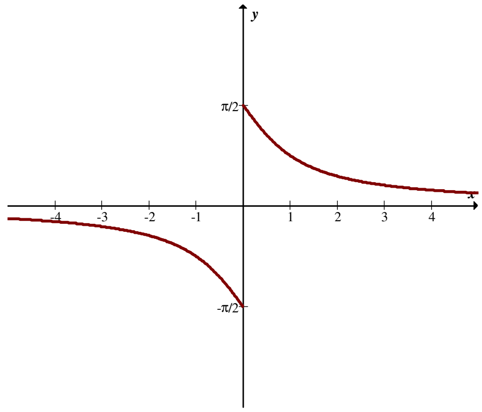
$$\left.\begin{aligned} \lim_{x \to 0^+} h(x) &= \lim_{x \to 0^+} \frac{x}{x} = 1 \\ \lim_{x \to 0^-} h(x) &= \lim_{x \to 0^-} \frac{x}{-x} = -1 \end{aligned}\right\} \Rightarrow \nexists \lim_{x \to 0} h(x), \text{ pois } \lim_{x \to 0^+} h(x) \neq \lim_{x \to 0^-} h(x).$$

7 – Determinar os limites à direita e à esquerda da função $f(x) = arcxtg\,\dfrac{1}{x}$ quando $x \to 0$.

Temos que:

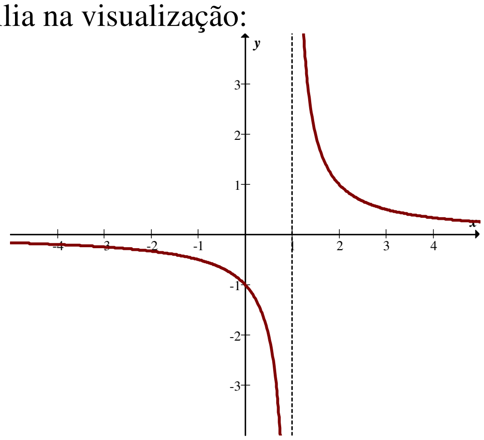
$$\lim_{x \to 0^+} = arc\,tg\,\frac{1}{x} = \frac{\pi}{2}$$
$$\lim_{x \to 0^-} = arc\,tg\,\frac{1}{x} = -\frac{\pi}{2}$$

O gráfico que segue ilustra esse exercício.



8 – Verifique se $\lim_{x\to 1} \frac{1}{x-1}$ existe.

O gráfico que segue auxilia na visualização:



Temos que:

$$\lim_{x\to 1^+} \frac{1}{x-1} = +\infty$$

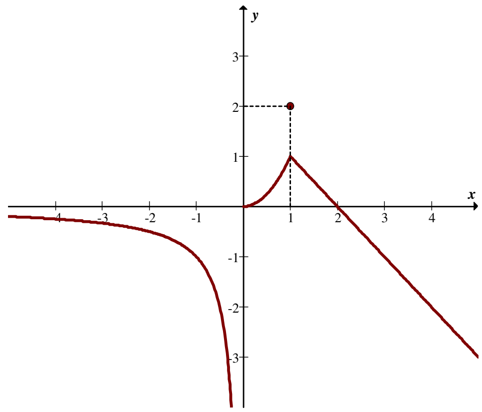
$$\lim_{x\to 1^-} \frac{1}{x-1} = -\infty$$

Segue que não existe o $\lim_{x\to 1} \frac{1}{x-1}$.

9 – Dada $f(x) = \begin{cases} \frac{1}{x} & , x < 0 \\ x^2 & , 0 \le x < 1 \\ 2 & , x = 1 \\ 2 - x & , x > 1 \end{cases}$.

Esboce o gráfico e calcule os limites indicados, se existirem:

Segue o gráfico



(a) $\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{1}{x} = -1$

(b) $\lim_{x\to 1} f(x) = \left\{ \begin{array}{l} \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2-x) = 1 \\ \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 = 1 \end{array} \right\} = 1$

(c) $\lim_{x\to 0^+} f(x) = 0$

(d) $\lim_{x\to 0^-} f(x) = -\infty$

(e) $\lim_{x\to 0} f(x) \nexists$

(f) $\lim_{x\to 2^+} f(x) = 0$

(g) $\lim_{x\to 2^-} f(x) = 0$

(h) $\lim_{x\to 2} f(x) = 0$

10 – Seja $f(x) = \dfrac{x^2-25}{x-5}$.

Calcule os limites indicados, se existirem:

(a) $\lim_{x\to 0} \dfrac{x^2-25}{x-5} = \lim_{x\to 0} \dfrac{(x-5)(x+5)}{(x-5)} = 5$

(b)

$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} \dfrac{(x-5)(x+5)}{(x-5)} = 10$

(c) $\lim_{x\to -5^-} f(x) = 0$

(d) $\lim_{x\to 5} f(x) = 10$

(e) $\lim_{x\to -5} f(x) = 0$