# 3.10 – EXERCÍCIO – pg. 83

1 – Para cada uma das seguintes funções, ache $\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}$:

(a) $f(x)=3x^2$

$$\lim_{x\to 2}\frac{3x^2-12}{x-2}=\lim_{x\to 2}\frac{(x-2)(3x+6)}{x-2}=12$$

(b) $f(x)=\dfrac{1}{x}$, $x\neq 0$

$$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}=\lim_{x\to 2}\frac{\frac{2-x}{2x}}{x-2}=\lim_{x\to 2}\frac{-(x-2)}{2x}\cdot\frac{1}{x-2}=\frac{-1}{4}$$

(c) $f(x)=\dfrac{2}{3}x^2$

$$\lim_{x\to 2}\frac{\frac{2}{3}x^2-\frac{8}{3}}{x-2}=\lim_{x\to 2}\frac{(x-2)\left(\frac{2}{3}x+\frac{4}{3}\right)}{x-2}=\frac{4}{3}+\frac{4}{3}=\frac{8}{3}$$

(d) $f(x)=3x^2+5x-1$

$$\lim_{x\to 2}\frac{3x^2+5x-1-21}{x-2}=\lim_{x\to 2}\frac{3x^2+5x-22}{x-2}=\lim_{x\to 2}\frac{(x-2)(3x+11)}{x-2}=17$$

(e) $f(x)=\dfrac{1}{x+1}$, $x\neq -1$

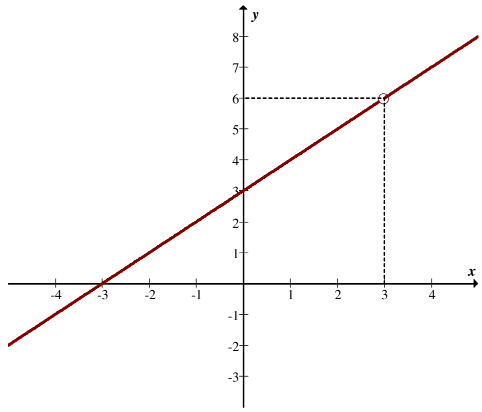
$$\lim_{x\to 2}\frac{\frac{1}{x+1}-\frac{1}{3}}{x-2}=\lim_{x\to 2}\frac{\frac{3-x-1}{3(x+1)}}{x-2}=\lim_{x\to 2}\frac{-2+x}{3(x+1)}\cdot\frac{-1}{x-2}=\frac{-1}{9}$$

(f) $f(x)=x^3$

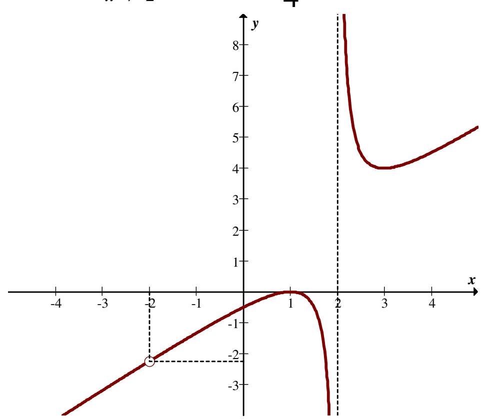
$$\lim_{x\to 2}\frac{x^3-8}{x-2}=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x-2}=12$$

2 - Esboçar o gráfico das seguintes funções e dar uma estimativa dos limites indicados

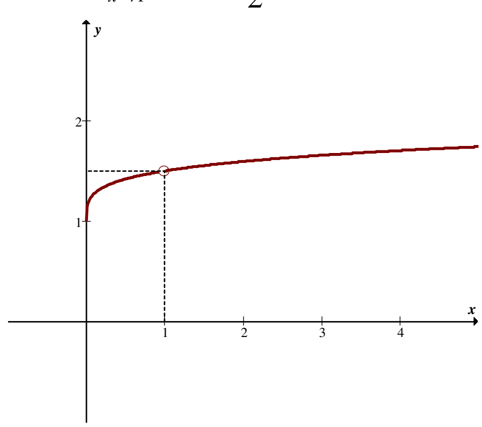
(a) $f(x)=\dfrac{x^2-9}{x-3}$;  $\lim\limits_{x\to 3} f(x)=6$



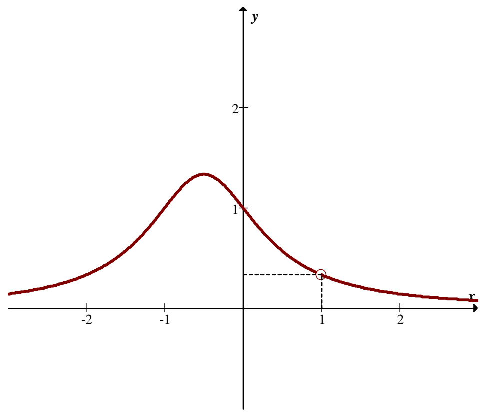

(b) $f(x)=\dfrac{x^3-3x+2}{x^2-4}$;  $\lim\limits_{x\to -2} f(x)=-\dfrac{9}{4}$.




(c) $f(x)=\dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1}$;  $\lim\limits_{x\to 1} f(x)=\dfrac{3}{2}$.




(d) $f(x)=\dfrac{x-1}{x^3-1}$;  $\lim\limits_{x\to 1} f(x)=\dfrac{1}{3}$.




3 – Calcular os limites indicados no Exercício 2 e comparar seus resultados com as estimativas obtidas.

(a) $\lim\limits_{x\to 3}\dfrac{x^2-9}{x-3}=\lim\limits_{x\to 3}\dfrac{(x-3)(x+3)}{x-3}=\lim\limits_{x\to 3}(x+3)=6.$

(b) $\lim\limits_{x\to -2}\dfrac{x^3-3x+2}{x^2-4}=\lim\limits_{x\to -2}\dfrac{(x+2)(x-1)^2}{(x+2)(x-2)}=\lim\limits_{x\to -2}\dfrac{(x-1)^2}{x-2}=-\dfrac{9}{4}.$

(c) $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1}=\lim\limits_{u\to 1}\dfrac{u^3-1}{u^2-1}=\lim\limits_{u\to 1}\dfrac{(u-1)(u^2+u+1)}{(u-1)(u+1)}=\lim\limits_{u\to 1}\dfrac{(u^2+u+1)}{(u+1)}=\dfrac{3}{2}.$

(d) $\lim\limits_{x\to 1}\dfrac{x-1}{x^3-1}=\lim\limits_{x\to 1}\dfrac{x-1}{(x-1)(x^2+x+1)}=\dfrac{1}{3}.$

Nos exercícios de 4 a 27 calcule os limites.

4 - $\lim\limits_{x\to -1}\dfrac{x^3+1}{x^2-1}=\lim\limits_{x\to -1}\dfrac{(x+1)(x^2-x+1)}{(x-1)(x+1)}=\dfrac{(-1)^2-(-1)+1}{-1-1}=-\dfrac{3}{2}.$

5 - $\lim\limits_{t\to -2}\dfrac{t^3+4t^2+4t}{(t+2)(t-3)}=\lim\limits_{t\to -2}\dfrac{(t+2)(t^2+2t)}{(t+2)(t-3)}=\dfrac{4-4}{-2-3}=\dfrac{0}{-5}=0.$

6 - $\lim\limits_{x\to 2}\dfrac{x^2+3x-10}{3x^2-5x-2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+5)}{(x-2)(3x+1)}=\lim\limits_{x\to 2}\dfrac{x+5}{3x+1}=\dfrac{2+5}{3\cdot 2+1}=\dfrac{7}{7}=1.$

7 - $\lim\limits_{t\to \frac{5}{2}}\dfrac{2t^2-3t-5}{2t-5}=\lim\limits_{t\to \frac{5}{2}}\dfrac{(2t-5)(t+1)}{(2t-5)}=\dfrac{5}{2}+1=\dfrac{7}{2}.$

8 - $\lim\limits_{x\to a}\dfrac{x^2+(1-a)x-a}{x-a}=\lim\limits_{x\to a}\dfrac{(x-a)(x+1)}{(x-a)}=a+1$.

9 - $\lim\limits_{x\to 4}\dfrac{3x^2-17x+20}{4x^2-25x+36}=\lim\limits_{x\to 4}\dfrac{(x-4)(3x-5)}{(x-4)(4x-9)}=\dfrac{12-5}{16-9}=\dfrac{7}{7}=1$.

10 - $\lim\limits_{x\to -1}\dfrac{x^2+6x+5}{x^2-3x-4}=\lim\limits_{x\to -1}\dfrac{(x+1)(x+5)}{(x+1)(x-4)}=\dfrac{-1+5}{-1-4}=-\dfrac{4}{5}$.

11 - $\lim\limits_{x\to -1}\dfrac{x^2-1}{x^2+3x+2}=\lim\limits_{x\to -1}\dfrac{(x-1)(x+1)}{(x+1)(x+2)}=\dfrac{-1-1}{-1+2}=\dfrac{-2}{1}=-2$.

12 - $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+2)}{(x-2)}=4$.

13 - $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2-12x+20}=\lim\limits_{x\to 2}\dfrac{(x-2)(x-3)}{(x-2)(x-10)}=\dfrac{2-3}{2-10}=\dfrac{-1}{-8}=\dfrac{1}{8}$.

14 - $\lim\limits_{h\to 0}\dfrac{(2+h)^4-16}{h}=\lim\limits_{h\to 0}\dfrac{h^4+8h^3+24h^2+32h+16-16}{h}=\lim\limits_{h\to 0}\dfrac{\cancel{h}(h^3+8h^2+24h+32)}{\cancel{h}}$

$=32$

15 - $\lim\limits_{t\to 0}\dfrac{(4+t)^2-16}{t}=\lim\limits_{t\to 0}\dfrac{16+8t+t^2-16}{t}=\lim\limits_{t\to 0}\dfrac{\cancel{t}(8+t)}{\cancel{t}}=8$.

16 - $\lim\limits_{t\to 0}\dfrac{\sqrt{25+3t}-5}{t}=\lim\limits_{t\to 0}\dfrac{25+3t-25}{t(\sqrt{25+3t}+5)}=\dfrac{3}{10}$.

17 - $\lim\limits_{t\to 0}\dfrac{\sqrt{a^2+bt}-a}{t}=\lim\limits_{t\to 0}\dfrac{a^2+bt-a^2}{t(\sqrt{a^2+bt}+a)}=\dfrac{b}{2a}$, $a>0$.

18 - $\lim\limits_{h\to 1}\dfrac{\sqrt{h}-1}{h-1}=\lim\limits_{h\to 1}\dfrac{h-1}{(h-1)(\sqrt{h}+1)}=\dfrac{1}{2}$.

19 - $\lim\limits_{h\to -4}\dfrac{\sqrt{2(h^2-8)}+h}{h+4}=\lim\limits_{h\to -4}\dfrac{2(h^2-8)-h^2}{(h+4)(\sqrt{2(h^2-8)}-h)}=\lim\limits_{h\to -4}\dfrac{2h^2-16-h^2}{(h+4)(\sqrt{2(h^2-8)}-h)}=$

$=\lim\limits_{h\to -4}\dfrac{(h+4)(h-4)}{(h+4)(\sqrt{2(h^2-8}-h)}=\dfrac{-8}{8}=-1$.

20 - $\lim_{h\to 0}\frac{\sqrt[3]{8+h}-2}{h}$

$$u^3 = 8+h \Rightarrow h = u^3 - 8$$

$$\lim_{u\to 2}\frac{u-2}{u^3-8} = \lim_{u\to 2}\frac{u-2}{(u-2)(u^2+2u+4)} = \frac{1}{12}.$$

21 - $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{-x} = \lim_{x\to 0}\frac{\cancel{1}+\cancel{x}-\cancel{1}}{-\cancel{x}(\sqrt{1+x}+1)} = \frac{-1}{2}.$

22 - $\lim_{x\to 0}\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b} = \lim_{x\to 0}\frac{(x^2+a^2-a^2)(\sqrt{x^2+b^2}+b)}{(x^2+b^2-b^2)(\sqrt{x^2+a^2}+a)} = \lim_{x\to 0}\frac{\cancel{2}b}{\cancel{2}a} = \frac{b}{a},\ a,b>0.$

23 - $\lim_{x\to a}\frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}$

Fazendo:
$x = u^3$
$a = b^3$
com $b \neq 0 \quad e \quad a \neq 0$ temos:

$$\lim_{u\to b}\frac{u-b}{u^3-b^3} = \lim_{u\to b}\frac{u-b}{(u-b)(u^2+bu+b^2)} = \frac{1}{b^2+b^2+b^2} = \frac{1}{3b^2}.$$

$$= \frac{1}{3\sqrt[3]{a^2}}.$$

24 - $\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$

Fazendo $x = u^{12}$ , $u \geq 0$ temos:

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1} = \lim_{u\to 1}\frac{u^4-1}{u^3-1} = \lim_{u\to 1}\frac{(u-1)(u^3+u^2+u+1)}{(u-1)(u^2+u+1)} = \frac{4}{3}.$$

25 - $\lim_{x\to 1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$

Fazendo $\sqrt[3]{x} = u$, temos:

$$\lim_{x\to 1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2} = \lim_{u\to 1}\frac{u^2-2u+1}{(u^3-1)^2} = \lim_{u\to 1}\frac{(u-1)(u-1)}{(u-1)^2(u^2+u+1)^2} = \frac{1}{9}.$$

26 - $\lim\limits_{x\to 4}\dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}=\lim\limits_{x\to 4}\dfrac{(9-5-x)(1+\sqrt{5-x}}{(1-5+x)(3+\sqrt{5+x}}=\lim\limits_{x\to 4}\dfrac{(4-x)(1+\sqrt{5-x})}{(-4+x)(3+\sqrt{5+x})}=\dfrac{-2}{6}=\dfrac{-1}{3}$ .

27 - $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}=\lim\limits_{x\to 0}\dfrac{1+x-1+x}{x(\sqrt{1+x)}+\sqrt{1-x})}=\lim\limits_{x\to 0}\dfrac{2x}{x(\sqrt{1+x)}+\sqrt{1-x})}=\dfrac{2}{2}=1$ .