5.14 – EXERCÍCIO – pg. 232

Determinar os seguintes limites com auxílio das regras de L´Hospital.

1 - $\lim\limits_{x\to 2}\dfrac{x^2-4x+4}{x^2-x-2}=\lim\limits_{x\to 2}\dfrac{2x-4}{2x-1}=\dfrac{0}{3}=0$.

2 - $\lim\limits_{x\to -1}\dfrac{x^2-1}{x^2+4x+3}=\lim\limits_{x\to -1}\dfrac{2x}{2x+4}=\dfrac{-2}{-2+4}=-1$.

3 - $\lim\limits_{x\to 0}\dfrac{x^2+6x}{x^3+7x^2+5x}=\lim\limits_{x\to 0}\dfrac{2x+6}{3x^2+14x+5}=\dfrac{6}{5}$.

4 - $\lim\limits_{x\to \frac{1}{2}}\dfrac{2x^2+x-1}{4x^2-4x+1}=\lim\limits_{x\to \frac{1}{2}}\dfrac{4x+1}{8x-4}=\dfrac{4\,\frac{1}{2}+1}{8\,\frac{1}{2}-4}=\dfrac{2+1}{4-4}=\dfrac{3}{0}=\infty$.

5 - $\lim\limits_{x\to 3}\dfrac{6-2x+3x^2-x^3}{x^4-3x^3-x+3}=\lim\limits_{x\to 3}\dfrac{-2+6x-3x^2}{4x^3-9x^2-1}=\dfrac{-11}{26}$.

6 - $\lim\limits_{x\to -1}\dfrac{x+1}{2x^4+2x^3+3x^2+2x-1}=\lim\limits_{x\to -1}\dfrac{1}{8x^3+6x^2+6x+2}=\dfrac{1}{-8+6-6+2}=-\dfrac{1}{6}$.

7 - $\lim\limits_{x\to \infty}\dfrac{x^2-6x+7}{x^3+7x-1}=\lim\limits_{x\to \infty}\dfrac{2x-6}{3x^2+7}=\lim\limits_{x\to \infty}\dfrac{2}{6x}=0$.

8 - $\lim\limits_{x\to -\infty}\dfrac{5-5x^3}{2-2x^3}=\lim\limits_{x\to -\infty}\dfrac{-15x^2}{-6x^2}=\dfrac{15}{6}=\dfrac{5}{2}$.

9 - $\lim\limits_{x\to +\infty}\dfrac{7x^5-6}{4x^2-2x+4}=\lim\limits_{x\to +\infty}\dfrac{35x^4}{8x-2}=\lim\limits_{x\to +\infty}\dfrac{140x^3}{8}=+\infty$.

10 - $\lim\limits_{x\to \infty}\dfrac{5-x+x^2}{2-x-2x^2}=\lim\limits_{x\to \infty}\dfrac{-1+2x}{-1-4x}=\lim\limits_{x\to \infty}\dfrac{2}{-4}=-\dfrac{1}{2}$.

11 - $\lim\limits_{x\to +\infty}\dfrac{e^x}{x^2}=\lim\limits_{x\to +\infty}\dfrac{e^x}{2x}=\lim\limits_{x\to +\infty}\dfrac{e^x}{2}=+\infty$.

12 - $\lim\limits_{x\to +\infty}\dfrac{x^{99}}{e^x}=\lim\limits_{x\to +\infty}\dfrac{99x^{98}}{e^x}=...=\lim\limits_{x\to +\infty}\dfrac{k}{e^x}=0$.

13 - $\lim\limits_{x\to 0}\dfrac{x}{e^x-\cos x}\quad\lim\limits_{x\to 0}\dfrac{1}{e^x+sen\,x}=\dfrac{1}{1+0}=1$.

14 - $\lim\limits_{x\to +\infty}x^2\left(e^{\frac{1}{x}}-1\right)\qquad 0\,.\,\infty\,?$

$$= \lim_{x\to+\infty}\left(\frac{e^{\frac{1}{x}}-1}{\frac{1}{x^2}}\right) = \lim_{x\to+\infty}\left(\frac{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{\frac{-2x}{x^4}}\right)$$

$$= \lim_{x\to+\infty} e^{\frac{1}{x}} \cdot \frac{1}{x^2} \cdot \frac{x^4}{2x} = \lim_{x\to+\infty} \frac{x\, e^{\frac{1}{x}}}{2} = +\infty$$

15 - $\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{(x-\pi/2)^2}$

$$= \lim_{x\to\frac{\pi}{2}} \frac{-\,sen\,x}{2\left(x-\frac{\pi}{2}\right)^2} = \frac{-1}{0} = \infty\,.$$

16 - $\lim_{x\to\infty} \frac{2^x}{2^x-1}$

$$= \lim_{x\to\infty} \frac{2^x \ln 2}{2^x \ln 2} = 1\,.$$

17 - $\lim_{x\to 2}\left(\frac{1}{2x-4}-\frac{1}{x-2}\right)$

$$= \lim_{x\to 2}\left(\frac{x-2-2x+4}{(2x-4)(x-2)}\right) = \lim_{x\to 2} \frac{-x+2}{(2x-4)(x-2)}$$

$$= \lim_{x\to 2} \frac{-x+2}{2x^2-8x+8} = \lim_{x\to 2} \frac{-1}{4x-8} = \frac{-1}{0} = \infty$$

18 - $\lim_{x\to+\infty}\left(\ln\frac{x}{x+1}\right)$

$$= \ln \lim_{x\to+\infty} \frac{x}{x+1} = \ln \lim_{x\to+\infty} \frac{1}{1} = \ln 1 = 0\,.$$

19 - $\lim_{x\to\pi/2}\left(\frac{x}{\cot g\, x}-\frac{\pi}{2\cos x}\right)$

$$= \lim_{x\to\pi/2}\left(\frac{2x\cos x-\pi\cot g\,x}{\cot g\,x\;2\cos x}\right)$$

$$= \lim_{x\to\pi/2}\left(\frac{2x\cos x-\pi\dfrac{\cos x}{senx}}{2\dfrac{\cos x}{senx}\cos x}\right)$$

$$= \lim_{x\to\pi/2}\left(\frac{2xsenx-\pi}{2\cos x}\right)= \lim_{x\to\pi/2}\left(\frac{2x\cos x+2senx}{-2senx}\right)=-1.$$

20 - $\lim\limits_{x\to+\infty} tgh\,x$

$$= \lim_{x\to+\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}} = \lim_{x\to+\infty}\frac{e^x-\dfrac{1}{e^x}}{e^x+\dfrac{1}{e^x}}$$

$$= \lim_{x\to+\infty}\frac{\dfrac{e^{2x}-1}{e^x}}{\dfrac{e^{2x}+1}{e^x}} = \lim_{x\to+\infty}\frac{e^{2x}-1}{e^{2x}+1}$$

$$= \lim_{x\to+\infty}\frac{e^{2x}.\,2}{e^{2x}.\,2}=1$$

21 - $\lim\limits_{x\to0}\dfrac{senh\,x}{sen\,x}$

$$\lim_{x\to0}\frac{\cosh x}{\cos x}=\frac{1}{1}=1$$

22 - $\lim\limits_{x\to\infty}\dfrac{\ln x}{\sqrt[3]{x}}$

$$=\lim_{x\to\infty}\frac{\dfrac{1}{x}}{\dfrac{1}{3}x^{\frac{-2}{3}}}=\lim_{x\to\infty}\frac{3\sqrt[3]{x^2}}{x}$$

$$=\lim_{x\to\infty}\frac{3\,\dfrac{2}{3}\,x^{\frac{-1}{3}}}{1}=\lim_{x\to\infty}\frac{2}{\sqrt[3]{x}}=0$$

23 - $\lim\limits_{x\to\pi/4}\dfrac{\sec^2 x-2tg\,x}{1+\cos 4x}$

$$= \lim_{x \to \pi/4} \frac{2\sec^2 x\, tg\, x - 2\sec^2 x}{-\,sen\, 4x.4}$$

$$= \lim_{x \to \pi/4} \frac{2\sec^2 x.\sec^2 x + tg^2 x.4\sec^2 x - 4\sec^2 x\, tg\, x}{-16\cos 4x} = \frac{8}{16} = \frac{1}{2}$$

24 - $\lim_{x \to 0} \frac{\cosh x - 1}{1 - \cos x}$

$$= \lim_{x \to 0} \frac{senh\, x}{+\,sen\, x} = \lim_{x \to 0} \frac{\cosh x}{\cos x} = \frac{1}{1} = 1$$

25 - $\lim_{x \to 0} (1 - \cos x)\cot g\, x$

$$= \lim_{x \to 0} \frac{1 - \cos x}{\frac{1}{\cot g\, x}} = \lim_{x \to 0} \frac{1 - \cos x}{tg\, x}$$

$$= \lim_{x \to 0} \frac{sen\, x}{\sec^2 x} = \frac{0}{1} = 0$$

26 - $\lim_{x \to 1} [\ln x \ln(x-1)]$

$$= \lim_{x \to 1} \frac{\ln(x-1)}{\frac{1}{\ln x}} = \lim_{x \to 1} \frac{\frac{1}{x-1}}{\frac{-\frac{1}{x}}{(\ln x)^2}}$$

$$= \lim_{x \to 1} \frac{1}{x-1} \cdot \frac{(\ln x)^2}{-\frac{1}{x}} = -\lim_{x \to 1} \frac{(\ln x)^2 x}{x-1}$$

$$= -\lim_{x \to 1} \frac{(\ln x)^2 1 + x\, 2\ln x \frac{1}{x}}{1} = -\lim_{x \to 1} (\ln x)^2 + 2\ln x = 0$$

27 - $\lim_{x \to 1} \left[\frac{1}{2\left(1 - \sqrt{x}\right)} - \frac{1}{3\left(1 - \sqrt[3]{x}\right)} \right]$

$$= \lim_{x\to 1}\left[\frac{3-3\sqrt[3]{x}-2+2\sqrt{x}}{6\left(1-\sqrt{x}\right)\left(1-\sqrt[3]{x}\right)}\right]$$

$$= \frac{1}{6}\lim_{x\to 1}\frac{1-3x^{1/3}+2x^{1/2}}{1-x^{1/3}-x^{1/2}+x^{5/6}}$$

$$= \frac{1}{6}\lim_{x\to 1}\frac{-x^{-2/3}+x^{-1/2}}{-\frac{1}{3}x^{-2/3}-\frac{1}{2}x^{-1/2}-\frac{5}{6}x^{-1/6}}$$

$$\frac{1}{6}\lim_{x\to 1}\frac{\frac{2}{3}x^{-5/3}-\frac{1}{2}x^{-3/2}}{\frac{2}{9}x^{-5/3}+\frac{1}{4}x^{-3/2}-\frac{5}{36}x^{-7/6}}$$

$$= \frac{1}{12}$$

28 - $\lim_{x\to 0^+} x^{\frac{3}{4+\ln x}}$

$$\ln \lim_{x\to 0^+} \ln x^{\frac{3}{4+\ln x}} = \lim_{x\to 0^+} \ln x^{\frac{3}{4+\ln x}}$$

$$= \lim_{x\to 0^+}\frac{3}{4+\ln x}\ln x$$

$$= \lim_{x\to 0^+}\frac{3\ln x}{4+\ln x} = \lim_{x\to 0^+}\frac{3\frac{1}{x}}{\frac{1}{x}} = 3 \quad \Rightarrow \quad \left[\lim_{x\to 0^+} x^{\frac{3}{4+\ln x}} = e^3\right]$$

29 - $\lim_{x\to 0^+} x^{sen\, x}$

$$\ln \lim_{x\to 0^+} x^{sen\,x} = \lim_{x\to 0^+} \ln x^{sen\,x}$$

$$= \lim_{x\to 0^+} sen\,x.\ln x = \lim_{x\to 0^+} \frac{\ln x}{\dfrac{1}{sen\,x}}$$

$$= \lim_{x\to 0^+} \frac{\dfrac{1}{x}}{-\cos\sec x.\cot g\,x} = \lim_{x\to 0^+} \frac{-1}{x\cos\sec x.\cot g\,x}$$

$$= \lim_{x\to 0^+} \frac{-sen^2 x}{x\cos x}$$

$$= \lim_{x\to 0^+} \frac{-2\,sen\,x\cos x}{-x\,sen\,x + \cos x} = \frac{0}{1} = 0$$

$$\therefore \quad \lim_{x\to 0^+} x^{sen\,x} = e^0 = 1$$

30 - $\lim\limits_{x\to 1} x^{\frac{1}{1-x}}$

$$\ln \lim_{x\to 1} \ln x^{\frac{1}{1-x}} = \lim_{x\to 1} \ln x^{\frac{1}{1-x}} = \lim_{x\to 1} \frac{1}{1-x} \ln x$$

$$= \lim_{x\to 1} \frac{\ln x}{1-x} = \lim_{x\to 1} \frac{\dfrac{1}{x}}{-1} = -1$$

$$\therefore \quad \lim_{x\to 1} x^{\frac{1}{1-x}} = e^{-1} = \frac{1}{e}$$

31 - $\lim\limits_{x\to 1^-} (1-x)^{\cos\frac{\pi x}{2}}$

$$\ln \lim_{x\to 1^-} (1-x)^{\cos\frac{\pi x}{2}} = \lim_{x\to 1^-} \ln(1-x)^{\cos\frac{\pi x}{2}}$$

$$= \lim_{x\to 1^-} \cos\frac{\pi x}{2} . \ln(1-x) = \lim_{x\to 1^-} \frac{\ln(1-x)}{\dfrac{1}{\cos\frac{\pi x}{2}}}$$

$$= \lim_{x\to 1^-} \frac{\dfrac{-1}{1-x}}{\sec\dfrac{\pi x}{2}\, tg\,\dfrac{\pi x}{2}} = \lim_{x\to 1^-} \frac{-1}{(1-x)\sec\dfrac{\pi x}{2}\, tg\,\dfrac{\pi x}{2}}$$

$$= \lim_{x\to 1^-} \frac{-1}{(1-x)\dfrac{1}{\cos\dfrac{\pi x}{2}} \cdot \dfrac{sen\dfrac{\pi x}{2}}{\cos\dfrac{\pi x}{2}}}$$

$$= \lim_{x\to 1^-} \frac{\cos^2\dfrac{\pi x}{2}}{(1-x)\, sen\dfrac{\pi x}{2}} = \lim_{x\to 1^-} \frac{-2\dfrac{\pi}{2}\cos\dfrac{\pi x}{2}\, sen\dfrac{\pi x}{2}}{\dfrac{\pi}{2}(1-x)\cos\dfrac{\pi x}{2} - sen\dfrac{\pi x}{2}} = 0$$

Assim, $\lim\limits_{x\to 1^-} (1-x)^{\cos\frac{\pi x}{2}} = e^0 = 1.$

32 - $\lim\limits_{x\to +\infty} x\, sen\, \pi / x$

$$= \lim_{x\to +\infty} \frac{sen\dfrac{\pi}{x}}{\dfrac{1}{x}} = \lim_{x\to +\infty} \frac{\left(\cos\dfrac{\pi}{x}\right)\dfrac{-\pi}{x^2}}{\dfrac{-1}{x^2}}$$

$$= \lim_{x\to +\infty} \pi \cos\frac{\pi}{x} = \pi$$

33 - $\lim\limits_{x\to \infty} \dfrac{x^{2/3}}{(x^2+2)^{1/3}}$

$$= \left(\lim_{x\to\infty} \frac{x^2}{x^2+2}\right)^{\frac{1}{3}} = \left(\lim_{x\to\infty} \frac{2x}{2x}\right)^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$$

34 - $\lim\limits_{x\to \infty} \dfrac{senh\, x}{x}$

$$= \lim_{x\to\infty} \frac{\cosh x}{1} = \infty$$

35 - $\lim\limits_{x\to \infty} (2x-1)^{2/x}$

$$\ln \lim_{x\to\infty} (2x-1)^{2/x} = \lim_{x\to\infty} \ln\, (2x-1)^{2/x}$$

$$= \lim_{x\to\infty} \frac{2}{x} \ln(2x-1) = \lim_{x\to\infty} \frac{2\ln(2x-1)}{x}$$

$$= \lim_{x\to\infty} \frac{2\dfrac{2}{2x-1}}{1} = \lim_{x\to\infty} \frac{4}{2x-1} = 0$$

$$\Rightarrow \lim_{x\to\infty} (2x-1)^{\frac{2}{x}} = e^0 = 1$$

36 - $\lim_{x\to 0}(\cos\ 2x)^{\frac{3}{x^2}}$

$$\ln\lim_{x\to 0}(\cos\ 2x)^{\frac{3}{x^2}}=\lim_{x\to 0}\ln(\cos\ 2x)^{\frac{3}{x^2}}$$

$$=\lim_{x\to 0}\frac{3}{x^2}\ln\ (\cos\ 2x)=\lim_{x\to 0}\frac{3\,\frac{-sen\,2x.2}{\cos\ 2x}}{2x}$$

$$=\lim_{x\to 0}\frac{-6\,sen\,2x}{\cos\ 2x}\,\frac{1}{2x}=\lim_{x\to 0}\frac{-6.2\cos 2x}{2\,(\cos\ 2x)+2x(-sen\,2x)\,2}$$

$$=\lim_{x\to 0}\frac{-12\cos\ 2x}{2\cos\ 2x-4x\ sen\,2x}=\frac{-12}{2-0}=-6$$

$$\Rightarrow\quad\lim_{x\to 0}(\cos\ 2x)^{\frac{3}{x^2}}=e^{-6}=\frac{1}{e^6}$$

37 - $\lim_{x\to 0^+}\frac{\ln(sen\,a\,x)}{\ln(sen\ x)}$

$$=\lim_{x\to 0^+}\frac{\frac{a\cos ax}{sen\ ax}}{\frac{\cos x}{sen\ x}}=\lim_{x\to 0^+}\frac{a\cos ax\ sen\ x}{sen\ ax\cos x}$$

$$=\lim_{x\to 0^+}\frac{a\cos ax\ \cos x-a^2 sen\ x.sen\ ax}{-sen\ ax.sen\ x+a\cos x.\cos ax}$$

$$=\lim_{x\to 0^+}\frac{a-0}{0+a}=1$$

38 - $\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{5}{x^2-x-6}\right)$

$$=\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{5}{(x-3)(x+2)}\right)=\lim_{x\to 3}\left(\frac{x+2-5}{(x-3)(x+2)}\right)=\lim_{x\to 3}\frac{x-3}{(x-3)(x+2)}=\frac{1}{5}$$

39 - $\lim_{x\to 0^+}\frac{1}{x^{tgx}}$

$$\ln\lim_{x\to 0^+}x^{-tgx}=\lim_{x\to 0^+}\ln x^{-tgx}=\lim_{x\to 0^+}-tgx\ln x=\lim_{x\to 0^+}-\frac{\ln x}{\frac{1}{tg\ x}}$$

$$=\lim_{x\to 0^+}-\frac{\ln x}{\cot g\ x}=\lim_{x\to 0^+}\frac{1}{x\cos\sec^2 x}=\lim_{x\to 0^+}\frac{1}{\frac{x}{sen^2 x}}$$

$$= \lim_{x\to 0^+} \frac{sen^2 x}{x} = \lim_{x\to 0^+} \frac{2\ sen\ x \cos x}{1} = \frac{0}{1} = 0$$

$$\Rightarrow \quad \lim_{x\to 0^+} \frac{1}{x^{tgx}} = e^0 = 1$$

40 - $\lim_{x\to 0^+} x^{\frac{2}{2+\ln x}}$

$$\ln \lim_{x\to 0^+} x^{\frac{2}{2+\ln x}} = \lim_{x\to 0^+} \ln x^{\frac{2}{2+\ln x}} = \lim_{x\to 0^+} \frac{2}{2+\ln x} \ln x = \lim_{x\to 0^+} \frac{2\ln x}{2+\ln x} = \lim_{x\to 0^+} \frac{2\frac{1}{x}}{\frac{1}{x}} = 2$$

$$\Rightarrow \quad \lim_{x\to 0^+} x^{\frac{2}{2+\ln x}} = e^2$$

41 - $\lim_{x\to \frac{\pi}{4}} (1 - tg\ x)\sec 2x$

$$= \lim_{x\to \frac{\pi}{4}} \frac{1 - tg\ x}{\frac{1}{\sec 2x}} = \lim_{x\to \frac{\pi}{4}} \frac{1 - tg\ x}{\cos 2x}$$

$$= \lim_{x\to \frac{\pi}{4}} \frac{\sec^2 x}{-2\ sen\ 2x} = \frac{-\left(\frac{2}{\sqrt{2}}\right)^2}{-2} = \frac{4}{2} \cdot \frac{1}{2} = 1$$

42 - $\lim_{x\to \infty} \frac{x\ \ln\ x}{x + \ln x}$

$$= \lim_{x\to \infty} \frac{x\ \frac{1}{x} + \ln\ x}{1 + \frac{1}{x}} = \lim_{x\to \infty} \frac{1 + \ln x}{1 + \frac{1}{x}} = \infty$$

43 - $\lim_{x\to 0} (e^x + x)^{1/x}$

$$\ln \lim_{x\to 0} (e^x + x)^{1/x} = \lim_{x\to 0} \ln(e^x + x)^{1/x} = \lim_{x\to 0} \frac{1}{x} \ln\ (e^x + x) = \lim_{x\to 0} \frac{\ln\ (e^x + x)}{x}$$

$$= \lim_{x\to 0} \frac{\frac{e^x + 1}{e^x + x}}{1} = \lim_{x\to 0} \frac{e^x + 1}{e^x + x}$$

$$= \frac{1+1}{1+0} = \frac{2}{1} = 2$$

$$\Rightarrow \quad \lim_{x\to 0} (e^x + x)^{\frac{1}{x}} = e^2$$