CAPÍTULO 6

6.2 – EXERCÍCIOS – pg. 246

Nos exercícios de 1 a 10, calcular a integral e, em seguida, derivar as respostas para conferir os resultados.

1. $\int \frac{dx}{x^3}$

$$\int x^{-3} dx = \frac{x^{-2}}{-2} + c = \frac{-1}{2x^2} + c$$

$$\frac{d}{dx}\left(\frac{-1}{2x^2}\right) = \frac{1}{x^3}$$

2. $\int \left(9t^2 + \frac{1}{\sqrt{t^3}}\right) d$

$$\int (9t^2 + t^{\frac{-3}{2}}) dt = 9.\frac{t^3}{3} + \frac{t^{\frac{-1}{2}}}{\frac{-1}{2}} + c = 3t^3 - \frac{2}{\sqrt{t}} + c.$$

$$\frac{d}{dt}\left(3t^3 - \frac{2}{\sqrt{t}} + c\right) = 9t^2 - 2.-\frac{1}{2}.t^{\frac{-3}{2}} = 9t^2 + \frac{1}{\sqrt{t^3}}.$$

3. $\int (ax^4 + bx^3 + 3c) dx$

$$= a\frac{x^5}{5} + b\frac{x^4}{4} + 3cx + C.$$

$$\frac{d}{dx}\left(\frac{a}{5}x^5 + \frac{b}{4}x^4 + 3cx + C\right) = \frac{a}{5}5x^4 + \frac{b}{4}4x^3 + 3c = ax^4 + bx^3 + 3c.$$

4. $\int \left(\frac{1}{\sqrt{x}} + \frac{x\sqrt{x}}{3}\right) dx$

$$=\int\left(x^{-\frac{1}{2}}+\frac{1}{3}x^{\frac{3}{2}}\right)dx=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\frac{1}{3}.\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+c=2\sqrt{x}+\frac{2}{15}x^{\frac{5}{2}}+c.$$

$$\frac{d}{dx}\left(2\sqrt{x}+\frac{2}{15}x^{\frac{5}{2}}+c\right)=2\frac{1}{2}x^{\frac{-1}{2}}+\frac{2}{15}.\frac{5}{2}.x^{\frac{3}{2}}=\frac{1}{\sqrt{x}}+\frac{x\sqrt{x}}{3}$$

5. $\int(2x^2-3)^2\,dx$

$$\int\left(4x^4-12x^2+9\right)dx=4\frac{x^5}{5}-12\frac{x^3}{3}+9x+c=\frac{4}{5}x^5-4x^3+9x+c.$$

$$\frac{d}{dx}\left(\frac{4}{5}x^5-4x^3+9x+c\right)=\frac{4}{5}5x^4-12x^2+9+c=4x^4-12x^2+9+c.$$

6. $\int\frac{dx}{sen^2x}$

$$=\int sen^{-2}x\,dx$$

$$=\int \cos\sec^2 x\,dx=-\cot g\ x+c.$$

$$\frac{d}{dx}\left(-\cot g\ x+c\right)=\cos\sec^2 x=\frac{1}{sen^2x}.$$

7. $\int\left(\sqrt{2y}-\frac{1}{\sqrt{2y}}\right)dy$

$$= \int \left(\sqrt{2}\,.\,y^{\frac{1}{2}} - \frac{1}{\sqrt{2}}.\,y^{\frac{-1}{2}} \right) dy = \frac{\sqrt{2}\;y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}}.\frac{y^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2\sqrt{2}}{3}\;y^{\frac{3}{2}} - \frac{2}{\sqrt{2}}.\,y^{\frac{1}{2}} + c = \sqrt{2y}\left(\frac{2}{3}y - 1\right) + c.$$

$$\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}\;y^{\frac{3}{2}} - \frac{2}{\sqrt{2}}.\,y^{\frac{1}{2}} + c \right) = \frac{2\sqrt{2}}{3}\,.\,\frac{3}{2}\;y^{\frac{1}{2}} - \frac{2}{\sqrt{2}}.\frac{1}{2}\,y^{-\frac{1}{2}} = \sqrt{2}\;y^{\frac{1}{2}} - \frac{1}{\sqrt{2}}\,y^{-\frac{1}{2}}$$

$$= \sqrt{2y} - \frac{1}{\sqrt{2y}}.$$

8. $\displaystyle\int \frac{\sqrt{2}\;dt}{3t^2+3} = \frac{\sqrt{2}}{3} arc\; tg\; t + c$

$$\frac{d}{dt}\left(\frac{\sqrt{2}}{3} arc\,tg\; t + c \right) = \frac{\sqrt{2}}{3}.\frac{1}{1+t^2} = \frac{\sqrt{2}}{3t^2+3}.$$

9. $\displaystyle\int x^3\sqrt{x}\;dx$

$$\int x^{\frac{7}{2}} dx = \frac{x^{\frac{9}{2}}}{\frac{9}{2}} + c$$

$$\frac{d}{dx}\left(\frac{2}{9}\,x^{\frac{9}{2}} + c \right) = \frac{2}{9}\,.\,\frac{9}{2} x^{\frac{7}{2}} = x^3\sqrt{x}$$

10. $\displaystyle\int \frac{x^5 + 2x^2 - 1}{x^4} dx$

$$= \int \left(x + 2x^{-2} - x^{-4} \right) dx = \frac{x^2}{2} + 2\,\frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + c = \frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + c$$

$$\frac{d}{dx}\left(\frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + c \right) = \frac{2x}{2} - \frac{-2}{x^2} + \frac{-1\,.\,9x^2}{9x^6} = x + \frac{2}{x^2} - \frac{1}{x^4} = \frac{x^5 + 2x^2 - 1}{x^4}.$$

Nos exercícios de 11 a 31, calcular as integrais indefinidas.

11. $\int \frac{x^2}{x^2+1}dx$

$$= \int \left(1 - \frac{1}{x^2+1}\right)dx = x - arc\,tg\; x + c.$$

12. $\int \frac{x^2+1}{x^2}dx$

$$\int (1 + x^{-2})dx = x + \frac{x^{-1}}{-1} + c = x - \frac{1}{x} + c.$$

13. $\int \frac{sen\; x}{\cos^2 x}dx$

$$= \int \frac{sen\, x}{\cos x} \cdot \frac{1}{\cos x}dx = \int tg\; x \,.\, \sec x\; dx = \sec x + c$$

14. $\int \sqrt{\frac{9}{1-x^2}}dx$

$$= \int \frac{3}{\sqrt{1-x^2}}dx = 3arc\; sen\, x + c.$$

15. $\int \sqrt{\frac{4}{x^4 - x^2}}dx$

$$\int \frac{2}{x\sqrt{x^2-1}}dx = 2arc\,\sec\; x + c.$$

16. $\int \frac{8x^4 - 9x^3 + 6x^2 - 2x + 1}{x^2}dx$

$$= \int (8x^2 - 9x + 6 - 2x^{-1} + x^{-2})dx$$

$$= \frac{8x^3}{3} - \frac{9x^2}{2} + 6x - 2\ln|x| - \frac{1}{x} + c$$

17. $\int\left(\frac{e^t}{2}+\sqrt{t}+\frac{1}{t}\right)dt$

$$=\frac{1}{2}e^t+\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\ln|t|+c=\frac{1}{2}e^t+\frac{2}{3}t^{\frac{3}{2}}+\ln|t|+c.$$

18. $\int\cos\theta\,.\,tg\,\theta\,d\theta$

$$=\int\cos\theta\,.\,\frac{sen\,\theta}{\cos\theta}\,d\theta=\int sen\,\theta d\theta=-\cos\theta+c.$$

19. $\int(e^x-e^{-x})dx$

$$=\int 2senh\,x\,dx=2\cosh x+c.$$

20. $\int(t+\sqrt{t}+\sqrt[3]{t}+\sqrt[4]{t}+\sqrt[5]{t})dt$

$$=\frac{t^2}{2}+\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}+\frac{t^{\frac{5}{4}}}{\frac{5}{4}}+\frac{t^{\frac{6}{5}}}{\frac{6}{5}}+c$$

$$=\frac{t^2}{2}+\frac{2}{3}t^{\frac{3}{2}}+\frac{3}{4}t^{\frac{4}{3}}+\frac{4}{5}t^{\frac{5}{4}}+\frac{5}{6}t^{\frac{6}{5}}+c$$

21. $\int\frac{x^{-1/3}-5}{x}dx$

$$=\int\left(x^{\frac{-4}{3}}-\frac{5}{x}\right)dx$$

$$=\frac{x^{-\frac{1}{3}}}{-\frac{1}{3}}-5\ln|x|+c=-3x^{-\frac{1}{3}}-5\ln|x|+c.$$

22. $\int 2^t-\sqrt{2}e^t+\cosh t)dt$

$$=\frac{2^t}{\ln 2}-\sqrt{2}\,e^t+senh\,t+c.$$

23. $\int\sec^2 x(\cos^3 x+1)dx$

$$=\int\left(\frac{1}{\cos^2 x}\,.\cos^3 x+\sec^2 x\right)dx = senx+tgx+c.$$

24. $\int\frac{dx}{(ax)^2+a^2}$, $a\neq 0,$ constante

$$=\int\frac{dx}{a^2x^2+a^2}=\int\frac{dx}{a^2(x^2+1)}=\frac{1}{a^2}arc\,tg\,x+c$$

25. $\int\frac{x^2-1}{x^2+1}dx$

$$=\int\left(1-\frac{2}{x^2+1}\right)dx = x-2arc\,tg\,x+c.$$

26. $\int\sqrt[3]{8(t-2)^6\left(t+\frac{1}{2}\right)^3}\,dt$

$$=\int 2(t-2)^2\left(t+\frac{1}{2}\right)dt=\int 2\left(t^2-4t+4\right)\left(t+\frac{1}{2}\right)dt$$

$$=2\int\left(t^3+\frac{1}{2}t^2-4t^2-2t+4t+2\right)dt=2\int\left(t^3-\frac{7}{2}t^2+2t+2\right)dt$$

$$=2\left(\frac{t^4}{4}-\frac{7}{2}.\frac{t^3}{3}+2\frac{t^2}{2}+2t\right)+c=\frac{t^4}{2}-\frac{7t^3}{3}+2t^2+4t+c.$$

27. $\int\left(e^t-\sqrt[4]{16t}+\frac{3}{t^3}\right)dt$

$$=e^t-2\frac{t^{\frac{5}{4}}}{\frac{5}{4}}+3\frac{t^{-2}}{-2}+c=e^t-\frac{8}{5}\,t^{\frac{5}{4}}-\frac{3}{2}\,t^{-2}+c.$$

28. $\int\frac{\ln x}{x\ln x^2}dx$

$$=\int\frac{\ln x}{x\,2\ln x}dx=\frac{1}{2}\int\frac{dx}{x}=\ln|x|+c.$$

29. $\int tg^2x\cos ec^2x\,dx$

$$=\int\frac{sen^2x}{\cos^2 x}\,\frac{1}{sen^2x}dx=\int\sec^2 x\,dx=tg\,x+c.$$

30. $\int (x-1)^2 (x+1)^2 \, dx$

$$= \int \left(x^2 - 2x + 1\right)\left(x^2 + 2x + 1\right) dx$$
$$= \int \left(x^4 + 2x^3 + x^2 - 2x^3 - 4x^2 - 2x + x^2 + 2x + 1\right) dx$$
$$= \int \left(x^4 - 2x^2 + 1\right) dx = \frac{x^5}{5} - 2\frac{x^3}{3} + x + c.$$

31. $\int \frac{dt}{\left(n - \frac{1}{2}\right) t^n}, \; onde \; n \in z$

Se n = 0, $\int -2dt = -2t + c$

Se n = 1, $\int \frac{dt}{\left(n - \frac{1}{2}\right) t^n} = \int \frac{2 \, dt}{t} = 2 \ln |t| + c$

Se n ≠ 1, $\frac{1}{\left(n - \frac{1}{2}\right)} \int t^{-n} \, dt = \frac{1}{\left(n - \frac{1}{2}\right)} \cdot \frac{t^{1-n}}{(1-n)} + c$

32. Encontrar uma primitiva F, da função $f(x) = x^{2/3} + x$, que satisfaça $F(1) = 1$.

$$F(x) = \int \left(x^{\frac{2}{3}} + x\right) dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^2}{2} + c$$

$$F(x) = \frac{3}{5} x^{\frac{5}{3}} + \frac{x^2}{2} + c$$
$$F(1) = \frac{3}{5} + \frac{1}{2} + c = 1$$

$$c = 1 - \frac{3}{5} - \frac{1}{2} = \frac{10 - 6 - 5}{10} = \frac{-1}{10}$$

$$F(x) = \frac{3}{5} x^{\frac{5}{3}} + \frac{x^2}{2} - \frac{1}{10}.$$

33. Determinar a função $f(x)$ tal que

$$\int f(x)dx = x^2 + \frac{1}{2}\cos 2x + c$$

$$\frac{d}{dx}\left(x^2 + \frac{1}{2}\cos 2x + c \right) = 2x + \frac{1}{2}(-sen2x).2 = 2x - sen2x.$$

34. Encontrar uma primitiva da função $f(x) = \frac{1}{x^2} + 1$ que se anule no ponto $x = 2$.

$$F(x) = \int \left(\frac{1}{x^2} + 1 \right) dx = \int \left(x^{-2} + 1 \right) dx = \frac{x^{-1}}{-1} + x + c = -\frac{1}{x} + x + c$$

$$F(2) = -\frac{1}{2} + 2 + c$$

$$c = \frac{1}{2} - 2 = \frac{1-4}{2} = \frac{-3}{2}$$

$$F(x) = -\frac{1}{x} + x - \frac{3}{2}$$

35. Sabendo que a função $f(x)$ satisfaz a igualdade.

$\int f(x)dx = sen\ x - x\cos\ x - \frac{1}{2}x^2 + c,$ determinar $f(\pi/4)$.

$$\frac{d}{dx}\left(sen\,x - x\cos x - \frac{1}{2}x^2 + c \right) = \cos\ x - (x(-sen\,x) + \cos x) - \frac{1}{2}2x$$

$$= \cos x + x\,sen\,x - \cos x - x = x\ sen\,x - x = x(sen\,x - 1)$$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\left(sen\frac{\pi}{4} - 1 \right) = \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} - 1 \right) = \frac{\pi}{4}.\frac{\sqrt{2}-1}{2} = \frac{\pi\left(\sqrt{2}-2\right)}{8}.$$

36. Encontrar uma função f tal que $f'(x) + sen\ x = 0$ e $f(0) = 2$.

$$f'(x) + sen\,x = 0$$

$$f'(x) = -senx$$

$$\int -sen\,x\,dx = +\cos x + c$$

$$f(x) = +\cos x + c$$

$$f(0) = \cos 0 + c = 2$$

$$c = 2 - 1 = 1$$

$$\therefore \quad f(x) = \cos x + 1.$$