6.4 – EXERCÍCIOS – pg. 250

Calcular as integrais seguintes usando o método da substituição.

1. $\int (2x^2 + 2x - 3)^{10} (2x+1)\, dx$

$$Fazendo - se:$$
$$u = 2x^2 + 2x - 3$$
$$du = (4x+2)dx = 2(2x+1)dx$$
$$Temos:$$
$$\int (2x^2 + 2x - 3)^{10} (2x+1)\, dx = \frac{1}{2} \frac{(2x^2 + 2x - 3)^{11}}{11} + c.$$

2. $\int (x^3 - 2)^{1/7} x^2 dx$

$$Fazendo - se:$$
$$u = x^3 - 2$$
$$du = 3x^2 dx$$
$$Temos:$$
$$\int (x^3 - 2)^{1/7} x^2 dx = \frac{1}{3} \frac{\left(x^3 - 2\right)^{\frac{8}{7}}}{\frac{8}{7}} + c = \frac{7}{24} \left(x^3 - 2\right)^{\frac{8}{7}} + c.$$

3. $\int \frac{x\, dx}{\sqrt[5]{x^2 - 1}}$

$$\int (x^2 - 1)^{-\frac{1}{5}} x\, dx$$
$$Fazendo - se:$$
$$u = x^2 - 1$$
$$du = 2x\, dx$$
$$Temos:$$
$$\int \frac{x\, dx}{\sqrt[5]{x^2 - 1}} = \frac{1}{2} \frac{\left(x^2 - 1\right)^{4/5}}{\frac{4}{5}} + c = \frac{5}{8} (x^2 - 1)^{\frac{4}{5}} + c$$

4. $\int 5x\sqrt{4 - 3x^2}\, dx$

$$= \int 5x(4-3x^2)^{\frac{1}{2}}dx = \int 5x\left(4-3x^2\right)^{\frac{1}{2}}dx$$

Fazendo-se:

$u = 4-3x^2$

$du = -6x\,dx$

Temos:

$$\int 5x\sqrt{4-3x^2}\,dx = 5.\frac{-1}{6}\frac{\left(4-3x^2\right)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{-5}{9}\left(4-3x^2\right)^{\frac{3}{2}} + c.$$

5. $\int \sqrt{x^2+2x^4}\,dx$

$$= \int x\left(1+2x^2\right)^{\frac{1}{2}}dx$$

$$= \frac{1}{4}\frac{\left(1+2x^2\right)^{\frac{3}{2}}}{\frac{3}{2}} + c \qquad \text{Fazendo: } \begin{array}{l} u = 1+2x^2 \\ du = 4x\,dx \end{array}$$

$$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + c$$

6. $\int (e^{2t}+2)^{1/3}e^{2t}dt$

Fazendo-se:

$u = e^{2t}+2$

$du = 2e^{2t}\,dt$

Temos:

$$\int (e^{2t}+2)^{1/3}e^{2t}dt = \frac{1}{2}\frac{\left(e^{2t}+2\right)^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3}{8}\left(e^{2t}+2\right)^{\frac{4}{3}} + c.$$

7. $\int \frac{e^t\,dt}{e^t+4}$

$= \int \frac{du}{u} = \ln\left|e^t+4\right| + c$, sendo que $u = e^t+4$ e $du = e^t\,dt$.

8. $\int \frac{e^{1/x}+2}{x^2}dx$

$$= \int e^{\frac{1}{x}} \frac{1}{x^2} dx + \int 2x^{-2} dx = -e^{\frac{1}{x}} + 2.\frac{x^{-1}}{-1} + c = -e^{\frac{1}{x}} - \frac{2}{x} + c.$$

Considerando - se :

$u = e^{\frac{1}{x}}$

$du = e^{\frac{1}{x}}.\frac{-1}{x^2}.$

9. $\int tg\ x \sec^2 x\ dx$

$= \frac{tg^2 x}{2} + c$. considerando-se: $u = tg\ x$, $du = \sec^2 x\ dx$

10. $\int sen^4 x \cos\ x\ dx$

$= \frac{sen^5 x}{5} + c$ considerando-se: $u = sen\, x$, $du = \cos x\, dx$

11. $\int \frac{sen\ x}{\cos^5 x} dx$

$= \int \cos^{-5} x\ .\ sen\, x\, dx$

$= -\frac{\cos^{-4} x}{-4} = \frac{1}{4\cos^4 x} + c$ utilizando: $u = \cos x$, $du = -senxdx$

$= \frac{1}{4} \sec^4 x + c$

12. $\int \frac{2sen\, x - 5\cos x}{\cos x} dx$

$= 2\int \frac{sen\, x}{\cos x} - 5 \int dx$ utilizando: $u = \cos x$, $du = -senxdx$

$= -2 \ln |\cos x| - 5x + c$

13. $\int e^x \cos 2\ e^x dx$

$= \frac{1}{2} sen 2e^x + c.$

Considerando - se :

$u = 2e^x$

$du = 2e^x dx.$

14. $\int \frac{x}{2}\cos x^2 \, dx$

$$= \frac{1}{2}\frac{1}{2} sen\, x^2 + c = \frac{1}{4} sen\, x^2 + c$$

Considerando - se :

$u = x^2$

$du = 2x\,dx.$

15. $\int sen\,(5\theta - \pi)d\theta$

$$= -\frac{1}{5}\cos(5\theta - \pi) + c.$$

Considerando - se :

$u = 5\theta - \pi$

$du = 5d\theta.$

16. $\int \frac{arc\ sen\ y}{2\sqrt{1-y^2}}dy$

$$= \frac{1}{2}\frac{(arc\, sen\, y)^2}{2} + c = \frac{1}{4}(arc\, sen\, y)^2 + c.$$

Considerando - se :

$u = arc\, sen\, y$

$du = \frac{1}{\sqrt{1-y^2}}dy.$

17. $\int \frac{2\sec^2\theta}{a + b\ tg\ \theta}d\theta$

$$= 2.\frac{1}{b}\ln|\,a + b\,tg\,\theta\,| + C$$

Considerando-se:

$u = a + b\,tg\,\theta$

$du = b.\sec^2\theta\, d\theta$

18. $\int \frac{dx}{16 + x^2}$

$$= \frac{1}{16}\int \frac{dx}{1+\left(\frac{x}{4}\right)^2} = \frac{1}{16} 4\ arc\,tg\,\frac{x}{4} + c = \frac{1}{4} arc\,tg\,\frac{x}{4} + c$$, utilizando: $u = \frac{x}{4}$, $du = \frac{1}{4}dx$

19. $\int \frac{dy}{y^2 - 4y + 4}$

$$= \int \frac{dy}{(y-2)^2} = \int (y-2)^{-2} dy = \frac{(y-2)^{-1}}{-1} + c = \frac{1}{2-y} + c$$, utilizando: $u = y - 2$, $du = dy$

20. $\int \sqrt[3]{sen\,\theta} \cos\theta\, d\theta$

$$= \int (sen\theta)^{1/3} \cos\theta\, d\theta = \frac{(sen\theta)^{4/3}}{\frac{4}{3}} + c = \frac{3}{4} sen^{\frac{4}{3}}\theta + c.$$

21. $\int \frac{\ln x^2}{x} dx$

$$\frac{1}{2}\frac{(\ln x^2)^2}{2} + c = \frac{1}{4}(\ln x^2)^2 + c = \frac{1}{4} 4(\ln x)^2 + c = (\ln x)^2 + c.$$

Considerando - se :

$u = \ln x^2$

$du = \frac{2x}{x^2} = \frac{2}{x}dx.$

22. $\int (e^{ax} + e^{-ax})^2\, dx$

$$= \int (e^{2ax} + 2 + e^{-2ax})^2 dx = \frac{1}{2a}e^{2ax} + 2x - \frac{1}{2a}e^{-2ax} + c$$

$$= \frac{1}{2a}(e^{2ax} - e^{-2ax}) + 2x + c = \frac{sen\,h\; 2ax}{a} + 2x + c.$$

23. $\int \sqrt{3t^4 + t^2}\, dt$

$$= \int \sqrt{t^2\left(3t^2+1\right)}dt = \int t\left(3t^2+1\right)^{\frac{1}{2}}dt = \frac{1}{6}\frac{\left(3t^2+1\right)^{\frac{3}{2}}}{\frac{3}{2}}+c$$

$$= \frac{1}{6}.\frac{3}{2}.\left(3t^2+1\right)^{\frac{3}{2}}+c = \frac{1}{9}.\left(3t^2+1\right)^{\frac{3}{2}}+c.$$

Considerando-se:

$$\begin{aligned} u &= 3t^2+1 \\ du &= 6t\ dt \end{aligned}.$$

24. $\int \frac{4\,dx}{4x^2+20x+34}$

$$= \int \frac{dx}{\left(x+\frac{5}{2}\right)^2+\left(\frac{3}{2}\right)^2} = \frac{1}{\frac{3}{2}}\,arc\,tg\,\frac{\left(x+\frac{5}{2}\right)}{\frac{3}{2}}+c = \frac{2}{3}\,arc\,tg\,\frac{2\left(x+\frac{5}{2}\right)}{3}+c.$$

25. $\int \frac{3\,dx}{x^2-4x+1}$

$$= \int \frac{3dx}{\left(x-2\right)^2-3} = -3\int \frac{\frac{dx}{3}}{\frac{3}{3}-\frac{\left(x-2\right)^2}{3}} = -3\,\frac{1}{3}\int \frac{dx}{1-\frac{\left(x-2\right)^2}{3}}$$

$$= -\sqrt{3}\,\frac{1}{2}\ln\left|\frac{1+\frac{x-2}{\sqrt{3}}}{1-\frac{x-2}{\sqrt{3}}}\right|+c = \frac{-\sqrt{3}}{2}\ln\left|\frac{x+\sqrt{3}-2}{\sqrt{3}+2-x}\right|+c.$$

Considerando-se:

$$u^2 = \frac{\left(x-2\right)^2}{3}$$

$$u = \frac{x-2}{\sqrt{3}}$$

$$du = \frac{1}{\sqrt{3}}dx$$

Resposta alternativa:

$arc\ tg\, h\, \frac{x-2}{\sqrt{3}} \quad se \quad \left|\frac{x-2}{\sqrt{3}}\right| < 1$

$arc\cot g\, h\, \frac{x-2}{\sqrt{3}} \quad se \quad \left|\frac{x-2}{\sqrt{3}}\right| > 1.$

26. $\int \frac{e^x dx}{e^{2x}+16}$

$= \frac{1}{4} arc\ tg\, \frac{e^x}{4} + c$

Considerando-se:

$u^2 = e^{2x}$

$u = e^x$

$du = e^{2x} dx$

27. $\int \frac{\sqrt{x+3}}{x-1} dx$

$$= \int \frac{u}{u^2-3-1}.2u\, du = \int \frac{2u^2}{u^2-4} du = 2\int\left(1+\frac{4}{u^2-4}\right) du = 2\left[u + 4\int \frac{du}{u^2-4}\right]$$

$$= 2u + 8\int \frac{\frac{-du}{4}}{\frac{4}{4}-\frac{u^2}{4}} = 2u - 2\int \frac{du}{1-\left(\frac{u}{2}\right)^2} = 2u - 2.2\,\frac{1}{2}\ln\left|\frac{1+\frac{u}{2}}{1-\frac{u}{2}}\right| + c$$

$$= 2u - 2\ln\left|\frac{2+u}{2-u}\right| + c = 2\sqrt{x+3} - 2\ln\left|\frac{2+\sqrt{x+3}}{2-\sqrt{x+3}}\right| + c.$$

Considerando-se:

$u^2 = x+3$

$x = u^2 - 3$

$dx = 2u\, du$

28. $\int \frac{3dx}{x \ln^2 3x}$

$$= \int (\ln 3x)^{-2} \frac{3dx}{x} = 3\frac{(\ln 3x)}{-1} + c = \frac{-3}{\ln 3x} + c.$$

Considerando-se:

$u = \ln 3x$

$du = \frac{3}{3x}\, dx$

29. $\int (sen\, 4x + \cos 2\pi)dx$

$$= \int sen\ 4x\ dx + \cos 2\pi \int dx = \frac{1}{4}(-\cos 4x) + x \cos 2\pi + c.$$

30. $\int 2^{x^2+1} x\ dx$

$$= \frac{1}{2}\frac{2^{x^2+1}}{\ln 2} + c = \frac{2^{x^2}}{\ln 2} + c.$$

Considerando-se:

$u = x^2 + 1$

$du = 2x\ dx$

31. $\int x\, e^{3x^2} dx$

$$= \frac{1}{6} e^{3x^2} + c$$

Considerando-se:

$u = 3x^2$

$du = 6x\ dx$

32. $\int \frac{dt}{(2+t)^2}$

$$= \int (2+t)^{-2} = \frac{(2+t)^{-1}}{-1} + c = \frac{-1}{2+t} + c\,.$$

Considerando-se:

$u = 2 + t$

$du = dt$

33. $\int \frac{dt}{t \ln t}$

$= \ln|\ln t| + c.$

Considerando-se: $u = \ln t$

$du = \frac{dt}{t}$

34. $\int 8x\sqrt{1-2x^2}\,dx$

$= 8\,\frac{-1}{4}\,\frac{(1-2x^2)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{-4}{3}(1-2x^2)^{\frac{3}{2}} + c.$

Considerando-se:

$u = 1-2x^2$

$du = -4x\,dx$

35. $\int (e^{2x}+2)^5 e^{2x}dx$

$= \frac{1}{2}\,\frac{(e^{2x}+2)^6}{6} + c = \frac{1}{12}(e^{2x}+2)^6 + c.$

Considerando-se:

$u = e^{2x}+2$

$du = 2e^{2x}dx$

36. $\int \frac{4t\,dt}{\sqrt{4t^2+5}}$

$= \int (4t^2-5)^{-\frac{1}{2}}4t\,dt$

$= \frac{1}{2}\,\frac{(4t^2+5)^{\frac{1}{2}}}{\frac{1}{2}} + c = \sqrt{4t^2+5} + c.$

Considerando-se:

$u = 4t^2+5$

$du = 8t\,dt$

37. $\int \frac{\cos x}{3 - sen\ x} dx$

$= -\ln|3 - sen\, x| + c$

Considerando-se:

$u = 3 - sen\ x$

$du = -\cos x\ dx$

38. $\int \frac{dv}{\sqrt{v}(1+\sqrt{v})^5}$

$= 2\,\frac{\left(1+\sqrt{v}\right)^{-4}}{-4} + c$

$= -\frac{1}{2\left(1+\sqrt{v}\right)^4} + c$

Considerando-se:

$u = 1 + \sqrt{v}$

$du = \frac{1}{2\sqrt{v}} dv$

39. $\int x^2 \sqrt{1+x} dx$

Considerando-se:

$1 + x = u^2$

$x = u^2 - 1 \quad \Rightarrow \quad dx = 2u\ du$

$\int x^2 \sqrt{1+x} dx = \int \left(u^2 - 1\right)^2 u\ 2u\ du = \int \left(u^4 - 2u^2 + 1\right) 2u^2\ du$

$= \int \left(2u^6 - 4u^4 + 2u^2\right) du = 2\,\frac{u^7}{7} - 4\,\frac{u^5}{5} + 2\,\frac{u^3}{3} + c$

$= \frac{2}{7}\sqrt{(1+x)^7} - \frac{4}{5}\sqrt{(1+x)^5} + \frac{2}{3}\sqrt{(1+x)^3} + c$

$= \frac{2}{7}(1+x)^3\sqrt{1+x} - \frac{4}{5}(1+x)^2\sqrt{1+x} + \frac{2}{3}(1+x)\sqrt{1+x} + c.$

40. $\int x^4 e^{-x^5} dx$

$$= \frac{-1}{5} e^{-x^5} + c$$

Considerando-se:

$$u = -x^5$$
$$du = -5x^4$$

41. $\int t \cos t^2 dt$

$$= \frac{1}{2}\, sen\, t^2 + c \text{, utilizando: } \begin{array}{l} u = t^2 \\ du = 2tdt \end{array}$$

42. $\int 8x^2 \sqrt{6x^3 + 5}\, dx$

$$= 8\, \frac{1}{18}\, \frac{(6x^3+5)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{4}{9}\, \frac{2}{3} (6x^3+5)^{\frac{3}{2}} + c = \frac{8}{27} (6x^3+5)^{\frac{3}{2}} + c.$$

Considerando-se:

$$u = 6x^3 + 5$$
$$du = 18x^2 dx$$

43. $\int sen^{1/2}\, 2\theta \cos 2\theta\, d\theta$

$$= \frac{1}{2}\, \frac{(sen\, 2\theta)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{1}{3} (sen 2\theta)^{3/2} + c\,.$$

Considerando-se:

$$u = sen\, 2\theta$$
$$du = 2 \cos 2\theta\, d\theta$$

44. $\int \sec^2 (5x + 3) dx$

$$= \frac{1}{5} tg\, (5x + 3) + c\,.$$

Considerando-se:

$u = 5x + 3$
$du = 5\,dx$

45. $\int \frac{sen\,\theta\, d\theta}{(5-\cos\theta)^3}$

$= \frac{(5-\cos\theta)^{-2}}{-2} + c$.

Considerando-se:

$u = 5 - \cos\theta$
$du = sen\,\theta\, d\theta$

46. $\int \cot g\ u\ du$

$= \int \frac{\cos u}{sen\,u} du = \ln|\,sen\,u\,| + c$

Considerando-se: $u = sen\,u$
$du = \cos\,u\,du$

47. $\int (1+e^{-at})^{3/2}\ e^{-at}\ dt, \quad a > 0$

$= \frac{-1}{a}\,\frac{(1+e^{-at})^{\frac{5}{2}}}{\frac{5}{2}} + c = -\frac{2}{5a}\,(1+e^{-at})^{\frac{5}{2}} + c.$

Considerando-se:

$u = 1 + e^{-at}$
$du = e^{-at}\,(-a)dt$

48. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

$= 2sen\sqrt{x} + c$.

Considerando-se:

$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}} dx$

49. $\int t\,\sqrt{t-4}\;dt$

$$= \int \left(u^2+4\right).u\;2u\;du = \int \left(2u^4+8u^2\right)du = 2\,\frac{u^5}{5} + 8\,\frac{u^3}{3} + c$$

$$= \frac{2}{5}\,\sqrt{(t-4)^5} + \frac{8}{3}\,\sqrt{(t-4)^3} + c$$

$$= \frac{2}{5}\,(t-4)^2\sqrt{t-4} + \frac{8}{3}\,(t-4)\sqrt{t-4} + c$$

Considerando-se:

$t-4 = u^2$

$t = u^2 + 4 \quad \Rightarrow \quad dt = 2u\;du$

50. $\int x^2\;(sen\;2x^3 + 4x)\;dx$

$$= \int x^2 sen\;2x^3 dx + \int 4x^3 dx = \frac{-1}{6}\cos\;2x^3 + 4\frac{x^4}{x} + c = \frac{-1}{6}\cos\;2x^3 + x^4 + c\,,$$

sendo que na primeira integral usamos:

$u = 2x^3$

$du = 6x^2 dx$