6.6 – EXERCÍCIOS – pg. 255

Resolver as seguintes integrais usando a técnica de integração por partes.

1. $\int x \, sen \, 5x \, dx$

$$u = x \Rightarrow du = dx$$

$$dv = sen 5x \, dx \quad \Rightarrow \quad v = \int sen 5x \, dx = \frac{-1}{5} \cos 5x$$

$$I = x \frac{-1}{5} \cos 5x - \int \frac{-1}{5} \cos 5x \, dx$$

$$= \frac{-x}{5} \cos 5x + \frac{1}{5} \cdot \frac{1}{5} sen 5x + c$$

$$= \frac{-x}{5} \cos 5x + \frac{1}{25} sen 5x + c$$

2. $\int \ln(1-x) dx$

$$u = \ln(1-x) \quad \Rightarrow \quad du = \frac{-1}{1-x} dx$$

$$dv = dx \quad \Rightarrow \quad v = x$$

$$I = \ln(1-x)x - \int x \frac{-1}{1-x} dx$$

$$I = x \ln(1-x) + \int \left(-1 + \frac{1}{1-x} \right) dx$$

$$I = x \ln(1-x) - x - \ln(1-x) + c$$

$$I = (x-1) \ln(1-x) - x + c$$

3. $\int t \, e^{4t} dt$

$$u = t \quad \Rightarrow \quad du = dt$$

$$dv = e^{4t} dt \quad \Rightarrow \quad v = \int e^{4t} dt = \frac{1}{4} e^{4t}$$

$$I = t\,\frac{1}{4}\,e^{4t} - \int \frac{1}{4}\,e^{4t}\;dt$$

$$= \frac{t}{4}\,e^{4t} - \frac{1}{4}\,.\,\frac{1}{4}\,e^{4t} + c$$

$$= e^{4t}\left(\frac{t}{4} - \frac{1}{16}\right) + c$$

4. $\int (x+1)\cos 2x dx$

$$u = x+1 \quad \Rightarrow \quad du = dx$$

$$dv = \cos 2x\;dx \quad \Rightarrow \quad v = \int \cos 2x\;dx = \frac{1}{2}\,sen\,2x$$

$$I = (x+1)\frac{1}{2}\,sen\,2x - \int \frac{1}{2}\,sen\,2x\;dx$$

$$= \frac{x+1}{2}\,sen\,2x + \frac{1}{4}\cos 2x + c$$

5. $\int x \ln 3x\; dx$

$$u = \ln 3x \quad \Rightarrow \quad du = \frac{3}{3x}dx$$

$$dv = xdx \quad \Rightarrow \quad v = \int x\,dx = \frac{x^2}{2}$$

$$I = (\ln 3x)\frac{x^2}{2} - \int \frac{x^2}{2}\,.\,\frac{1}{x}dx$$

$$= \frac{x^2}{2}\ln 3x - \frac{1}{2}\,.\,\frac{x^2}{2} + c$$

$$= \frac{x^2}{2}\left(\ln 3x - \frac{1}{2}\right) + c$$

6. $\int \cos^3 x\; dx$

$$u = \cos^2 x \quad \Rightarrow \quad du = -2\cos x\,.\,sen\,x\;dx$$

$$dv = \cos x\;dx \quad \Rightarrow \quad v = \int \cos x\;dx = sen\;x$$

$$I = \cos^2 x . sen\, x - \int sen\, x\, (-2) \cos x\; senx\, dx$$

$$= \cos^2 x . sen\, x + 2 \int sen^2 x \cos x\, dx$$

$$= \cos^2 x . sen\, x + 2\, \frac{sen^3 x}{3} + c$$

7. $\int e^x \cos \frac{x}{2}\, dx$

$$u = e^x \quad \Rightarrow \quad du = e^x dx$$

$$dv = \cos \frac{x}{2} dx \quad \Rightarrow \quad v = \int \cos \frac{x}{2} dx = 2 sen \frac{x}{2}$$

$$I = e^x\; 2\, sen \frac{x}{2} - \int 2\, sen \frac{x}{2}\, e^x dx$$

$$u = e^x \quad \Rightarrow \quad du = e^x dx$$

$$dv = sen \frac{x}{2} du \quad \Rightarrow \quad v = -2 \cos \frac{x}{2}$$

$$I = 2 e^x\, sen \frac{x}{2} - 2 \left[e^x (-2) \cos \frac{x}{2} - \int -2 \cos \frac{x}{2} e^x dx \right]$$

$$= 2 e^x sen \frac{x}{2} + 4 e^x \cos \frac{x}{2} - 4I$$

$$5I = 2 e^x sen \frac{x}{2} + 4 e^x \cos \frac{x}{2}$$

$$= \frac{1}{5} \left(2 e^x sen \frac{x}{2} + 4 e^x \cos \frac{x}{2} \right) + c$$

8. $\int \sqrt{x} \ln x\, dx$

$$u = \ln x \quad \Rightarrow \quad du = \frac{1}{x} dx$$

$$dv = \sqrt{x}\, dx \quad \Rightarrow \quad v = \int x^{\frac{1}{2}}\, dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$$

$$I = (\ln x).\frac{2}{3}x^{\frac{3}{2}} - \int \frac{2}{3}x^{\frac{3}{2}}.\frac{1}{x}\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}.x^{\frac{3}{2}} + c$$

9. $\int \cos ec^3 x\,dx$

$$u = \cos ec\,x \quad \Rightarrow \quad du = -\cos ec\,x\,.\cot g\,x\,.\,dx$$

$$dv = \cos ec^2 x\,dx \quad \Rightarrow \quad v = \int \cos ec^2\,x\,dx = -\cot g\,x$$

$$I = \cos ec\,x\,.(-\cot g\;x) - \int -\cot g\,x.(-\cos ec\,x).\cot g\,x\,dx$$

$$= -\cos ec\,x\,.\cot g\;x - \int \cot g^2\,x.\cos ec\,x\,dx$$

$$= -\cos ec\,x\,.\cot g\;x - \int \frac{\cos^2 x}{sen^2\,x}.\frac{1}{sen\,x}\,dx$$

$$= -\cos ec\,x\,.\cot g\;x - \int \frac{\cos^2 x}{sen^3\,x}\,dx$$

$$= -\cos ec\,x\,.\cot g\;x - \int sen^{-3}\,x\,.\cos x\,.\cos x\,dx$$

$$u = \cos x \quad \Rightarrow \quad du = -sen\,x\,dx$$

$$dv = sen^{-3}\,x\cos dx \quad \Rightarrow \quad v = \frac{sen^{-2}\,x}{-2}$$

$$I = -\cos ec\,x\,.\cot g\;x - \left[\cos x\,.\frac{sen^{-2}\,x}{-2} - \int \frac{sen^{-2}\,x}{-2}.(-sen\,x)\,dx\right]$$

$$I = -\cos ec\,x\,.\cot g\;x + \frac{\cos x}{2\,sen^2\,x} + \frac{1}{2}\int \cos ec\;x\,dx$$

$$I = -\cos ec\,x\,.\cot g\;x + \frac{\cos x}{2\,sen^2\,x} + \frac{1}{2} + c$$

$$I = -\cos ec\,x\,.\cot g\;x + \frac{1}{2}\cot g\;x\,.\cos ec\;x + \frac{1}{2}\ln\left|\cos ec\,x - \cot g\,x\right| + c$$

$$= -\frac{1}{2}\cos ec\,x\,.\cot g\;x + \frac{1}{2}\ln\left|\cos ec\,x - \cot g\,x\right| + c$$

10. $\int x^2\,\cos a\;x\,dx$

$$u = x^2 \quad \Rightarrow \quad du = 2x\,dx$$

$$dv = \cos ax\,dx \quad \Rightarrow \quad v = \int \cos ax\,dx = \frac{1}{a}\,sen\,ax$$

$$I = x^2 \,.\, \frac{1}{a}\,sen\,ax - \int \frac{1}{a}\,sen\,ax \,.\, 2x\,dx$$

$$I = \frac{x^2}{a}\,sen\,ax - \frac{2}{a}\int x sen\,ax \;\; dx$$

$$u = x \quad \Rightarrow \quad du = dx$$

$$dv = sen\;ax\;dx \quad \Rightarrow \quad v = \frac{-1}{a}\cos ax$$

$$I = \frac{x^2}{a}\,sen\,ax - \frac{2}{a}\left[x \,.\, \frac{-1}{a}\cos\,ax - \int \frac{-1}{a}\cos ax\,dx \right]$$

$$= \frac{x^2}{a}\,sen\,ax + \frac{2x}{a^2}\cos\,ax - \frac{2}{a^2}\,sen\;ax\,\frac{1}{a} + c$$

$$= \frac{x^2}{a}\,sen\,ax + \frac{2x}{a^2}\cos\,ax - \frac{2}{a^3}\,sen\;ax + c$$

11. $\int x \cos ec^2 \; x\,dx$

$$u = x \quad \Rightarrow \quad du = dx$$

$$dv = \cos\sec^2 x\,dx \Rightarrow \quad v = \int \cos\sec^2 x\,dx$$

$$v = -\cot g \;\, x$$

$$I = -x \,.\, \cot g - \int - \cot g \;.\, dx$$

$$= -x \,.\, \cot g + \ln\left| sen\;x \right| + c$$

12. $\int arc\,\cot g\;2x\,dx$

$$u = arc\,\cot g\;2x \quad \Rightarrow \quad du = \frac{-2}{1+4x^2}\,dx\,,$$

$$dv = dx \quad \Rightarrow \quad v = x$$

$$I = arc\,\cot g\;2x\;x - \int x\,\frac{-2}{1+4x^2}\,dx$$

$$= x\;arc\,\cot g\;2x + 2\int \frac{x\,dx}{1+4x^2}$$

$$= x\;arc\,\cot g\;2x + \frac{1}{4}\ln\left|1+4x^2\right| + c$$

13. $\int e^{ax}\ sen\ bx\ dx$

$$u = e^{ax} \quad \Rightarrow \quad du = a\,e^{ax}\ dx$$

$$dv = sen\ bx\ dx \quad \Rightarrow \quad v = \int sen\ bx\ dx = \frac{-1}{b}\cos bx$$

$$I = e^{ax}\ \frac{-1}{b}\cos bx - \int -\frac{1}{b}\cos bx\ a\,e^{ax}\ dx$$

$$I = \frac{-e^{ax}}{b}\cos bx + \frac{a}{b}\int e^{ax}\cos bx\ dx$$

$$u = e^{ax} \quad \Rightarrow \quad du = a\ e^{ax} dx$$

$$dv = \cos bx\ dx \quad \Rightarrow \quad v = \frac{1}{b}\ sen\ bx$$

$$I = \frac{-e^{ax}}{b}\cos bx + \frac{a}{b}\left[e^{ax}\ .\frac{1}{b}sen\ bx - \int \frac{1}{b}sen\ bx\ a\ e^{ax}\ dx\right]$$

$$I = \frac{-e^{ax}}{b}\cos bx + \frac{a}{b^2}e^{ax}\ sen\ bx - \frac{a^2}{b^2}I$$

$$I + \frac{a^2}{b^2}I = \frac{-e^{ax}}{b}\cos bx + \frac{a}{b^2}e^{ax}\ sen\ bx$$

$$I = \frac{b^2}{b^2+a^2}\left(\frac{-e^{ax}}{b}\cos bx + \frac{a}{b^2}e^{ax}\ sen\ bx\right)$$

$$I = \frac{-b\ e^{ax}\ \cos bx + a\ e^{ax}\ sen\ bx}{a^2+b^2} + c$$

14. $\int \frac{\ln(ax+b)}{\sqrt{ax+b}}\ dx$

$$u = \ln\left(ax+b\right) \quad \Rightarrow \quad du = \frac{a}{ax+b}\ dx$$

$$dv = \left(ax+b\right)^{-\frac{1}{2}} dx \quad \Rightarrow \quad v = \frac{1}{a}\ \frac{\left(ax+b\right)^{\frac{1}{2}}}{\frac{1}{2}}$$

$$I = \ln(ax+b) \, . \, \frac{2}{a}\sqrt{ax+b} - \int \frac{2}{a}(ax+b)^{\frac{1}{2}} \, \frac{a}{ax+b} \, dx$$

$$I = \frac{2}{a}\sqrt{ax+b}\,\ln(ax+b) - 2\int (ax+b)^{-\frac{1}{2}}\,dx$$

$$I = \frac{2}{a}\sqrt{ax+b}\,\ln(ax+b) - 2\,\frac{1}{a}\,\frac{(ax+b)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$I = \frac{2}{a}\sqrt{ax+b}\,\ln(ax+b) - \frac{4}{a}\sqrt{ax+b} + c$$

15. $\int x^3 \sqrt{1-x^2}\,dx$

$$u = x^2 \quad \Rightarrow \quad du = 2x\,dx$$

$$dv = (1-x^2)^{\frac{1}{2}}\,x\,dx \quad \Rightarrow \quad v = \frac{-1}{2}\,\frac{(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}}$$

$$I = x^2 \, . \, \frac{-1}{3}(1-x^2)^{\frac{3}{2}} - \int \frac{-1}{3}(1-x^2)^{\frac{3}{2}}\,2x\,dx$$

$$I = \frac{-1}{3}\,x^2\,(1-x^2)^{\frac{3}{2}} - \frac{1}{3}\,\frac{(1-x^2)^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$I = \frac{-1}{3}\,x^2\,(1-x^2)^{\frac{3}{2}} - \frac{2}{15}(1-x^2)^{\frac{5}{2}} + c$$

16. $\int \ln^3 2x\,dx$

$$u = \ln^3 2x \quad \Rightarrow \quad du = 3\ln^2 2x\,\frac{2}{2x}\,dx$$

$$dv = dx \quad \Rightarrow \quad v = x$$

$$I = \ln^3 2x\;x - \int x \, . \ln^2 2x\,\frac{dx}{x}$$

$$I = x\ln^3 2x - 3\int \ln^2 2x\,dx$$

$$u = \ln^2 2x \quad \Rightarrow \quad du = 2\,\ln 2x\,\frac{2}{2x}\,dx$$

$$dv = dx \quad \Rightarrow \quad v = x$$

$$I = x\ln^3 2x - 3\left[\ln^2 2x\,.x - \int x\, 2\ln 2x\,\frac{dx}{x}\right]$$

$$I = x\ln^3 2x - 3\left[x\ \ln^2 2x - 2\int \ln 2x\, dx\right]$$

$$u = \ln 2x \quad \Rightarrow \quad du = \frac{2}{2x}dx$$

$$dv = dx \quad \Rightarrow \quad v = x$$

$$I = x\ln^3 2x - 3\,x\ln^2 2x + 6\left[\ln 2x\ x - \int x\,\frac{dx}{x}\right]$$

$$I = x\ln^3 2x - 3x\ln^2 2x + 6x\ln 2x - 6x + c$$

17. $\int arc\ tg\ a\ x\ dx$

$$u = arc\ tg\ ax \quad \Rightarrow \quad du = \frac{a}{1+a^2x^2}dx$$

$$dv = dx \quad \Rightarrow \quad v = x$$

$$I = arc\ tg\ ax\ .\ x - \int x\,\frac{a}{1+a^2x^2}\,dx$$

$$I = x\ arc\ tg\ ax - \frac{1}{2a}\ln\left|1+a^2x^2\right| + c$$

18. $\int x^3 sen\ 4x\ dx$

$$u = x^3 \quad \Rightarrow \quad du = 3x^2dx$$

$$dv = sen\ 4x\ dx \quad \Rightarrow \quad v = \frac{-1}{4}\cos 4x$$

$$I = x^3\,\frac{-1}{4}\cos 4x - \int \frac{-1}{4}\cos 4x\ 3x^2\ dx$$

$$= \frac{-x^3}{4}\cos 4x + \frac{3}{4}\int x^2\cos 4x\ dx$$

$$u = x^2 \quad \Rightarrow \quad du = 2xdx$$

$$dv = \cos 4x\ dx \quad \Rightarrow \quad v = \frac{1}{4}sen\ 4x$$

$$I = \frac{-x^3}{4}\cos 4x + \frac{3}{4}\int x^2 \cos 4x \, dx$$

$$= \frac{-x^3}{4}\cos 4x + \frac{3}{4}\left[x^2 . \frac{1}{4} sen 4x - \int \frac{1}{4} sen 4x . 2x dx\right]$$

$$= \frac{-x^3}{4}\cos 4x + \frac{3}{16}x^2 sen 4x - \frac{3}{8}\int x sen 4x dx$$

$$u = x \quad \Rightarrow \quad du = dx$$

$$dv = sen\ 4x\ dx \quad \Rightarrow \quad v = -\frac{1}{4}\cos 4x$$

$$I = \frac{-x^3}{4}\cos 4x + \frac{3}{16}x^2 sen 4x - \frac{3}{8}\left[x . \frac{-1}{4}\cos 4x - \int \frac{-1}{4}\cos 4x dx\right]$$

$$= \frac{-x^3}{4}\cos 4x + \frac{3}{16}x^2 sen 4x + \frac{3}{32}x\cos 4x - \frac{3}{128} sen 4x + c$$

19. $\int (x-1)e^{-x} dx$

$$= \int x e^{-x} dx - \int e^{-x} dx$$

$$u = x \quad \Rightarrow \quad du = dx$$

$$dv = e^{-x} \quad \Rightarrow \quad v = -e^{-x}$$

$$I = -x\, e^{-x} - \int - e^{-x} dx - \int e^{-x} dx$$

$$= -x\, e^{-x} + c$$

20. $\int x^2 \ln x \, dx$

$$u = \ln x \quad \Rightarrow \quad du = \frac{1}{x} dx$$

$$dv = x^2 dx \quad \Rightarrow \quad v = \frac{x^3}{3}$$

$$I = \ln x \, \frac{x^3}{3} - \int \frac{x^3}{3} \frac{1}{x} \, dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\frac{x^3}{3} + c$$

21. $\int x^2 \, e^x \, dx$

$$u = x^2 \quad \Rightarrow \quad du = 2x\,dx$$

$$dv = e^x dx \quad \Rightarrow \quad v = e^x$$

$$I = x^2 e^x - \int e^x 2x\,dx$$

$$u = x \quad \Rightarrow \quad du = dx$$

$$dv = e^x du \quad \Rightarrow \quad v = e^x$$

$$I = x^2 e^x - 2\left[x\,e^x - \int e^x dx\right]$$

$$= x^2 e^x - 2x\,e^x + 2e^x + c$$

22. $\int arc\ sen\,\frac{x}{2}\,dx$

$$u = arc\ sen\,\frac{x}{2} \quad \Rightarrow \quad du = \frac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}}\,dx$$

$$dv = dx \quad \Rightarrow \quad v = x$$

$$I = arc\ sen\,\frac{x}{2}\,x - \int x\frac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}}\,dx$$

$$= x\ arc\ sen\,\frac{x}{2} + \frac{1}{2}\,2\,\frac{\left(1-\frac{x^2}{4}\right)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= x\ arc\ sen\,\frac{x}{2} + 2\left(\frac{4-x^2}{4}\right)^{\frac{1}{2}} + c$$

$$= x\ arc\ sen\,\frac{x}{2} + \sqrt{4-x^2} + c$$

23. $\int (x-1)\sec^2 x\,dx$

$$u = x-1 \quad \Rightarrow \quad du = dx$$

$$dv = \sec^2 x\,dx \quad \Rightarrow \quad v = tg\ x$$

$$I = (x-1)\ tg\ x - \int tg\ x\ dx$$
$$= (x-1) tg\ x + \ln|\cos x| + c$$

24. $\int e^{3x} \cos 4x\ dx$

$$u = e^{3x} \quad \Rightarrow \quad du = 3\ e^{3x} dx$$
$$dv = \cos 4x dx \Rightarrow v = \frac{1}{4} sen\ 4x$$

$$I = e^{3x}\ \frac{1}{4} sen\ 4x - \int \frac{1}{4} sen\ 4x\ 3e^{3x} dx$$

$$u = e^{3x} \Rightarrow du = 3e^{3x} dx$$
$$dv = sen 4x \Rightarrow v = \frac{-1}{4} \cos\ 4x$$

$$I = \frac{e^{3x}}{4} sen\ 4x - \frac{3}{4}\left[e^{3x}\ .\frac{-1}{4} \cos\ 4x - \int \frac{-1}{4} \cos\ 4x\ 3e^{3x} dx \right]$$
$$I = \frac{e^{3x}}{4} sen\ 4x + \frac{3}{16}\ e^{3x} \cos\ 4x - \frac{9}{16} I$$
$$I + \frac{9}{16} I = \frac{4e^{3x} sen\ 4x + 3e^{3x} \cos 4x}{16}$$
$$I = \frac{4e^{3x} sen\ 4x + 3e^{3x} \cos 4x}{25} + c$$

25. $\int x^n \ln\ x\ dx, \quad n \in N$

$$u = \ln x \quad \Rightarrow \quad du = \frac{1}{x} dx$$
$$dv = x^n dx \quad \Rightarrow \quad v = \frac{x^{n+1}}{n+1}$$

$$I = \ln x\ \frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1} . \frac{1}{x} dx$$
$$= \frac{x^{n+1}}{n+1} \ln\ x - \frac{1}{n+1}\ \frac{x^{x+1}}{n+1} + c$$
$$= \frac{x^{n+1}}{n+1} \ln\ x - \frac{x^{x+1}}{(n+1)^2} + c$$

26. $\int \ln(x^2+1)dx$

$$u = \ln\,(x^2+1) \quad \Rightarrow \quad du = \frac{2x}{x^2+1}dx$$

$$dv = dx \Rightarrow v = x$$

$$I = \ln\,(x^2+1)\;x - \int x\,\frac{2x}{x^2+1}dx$$

$$= x\ln\,(x^2+1) - 2\int\left(1 - \frac{1}{x^2+1}\right)dx$$

$$= x\ln\,(x^2+1) - 2\,(x - arc\ tg\ x) + c$$

$$= x\ln\,(x^2+1) - 2x + 2arc\ tg\ x + c$$

27. $\int \ln\left(x+\sqrt{1+x^2}\right)dx$

$$u = \ln\left(x+\sqrt{1+x^2}\right) \quad \Rightarrow \quad du = \frac{1+\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}2x}{x+\sqrt{1+x^2}}dx$$

$$dv = dx \Rightarrow v = x$$

$$I = \ln\left(x+\sqrt{1+x^2}\right)x - \int x\frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}dx$$

$$I = x\ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{2}\,\frac{\left(1+x^2\right)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= x\ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2} + c$$

28. $\int x\ arc\ tg\ x\ dx$

$$u = arc\ tg\ x \quad \Rightarrow \quad du = \frac{1}{1+x^2}dx$$

$$dv = x\,dx \quad \Rightarrow \quad v = \frac{x^2}{2}$$

$$I = arc\ tg\ x\ \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2}\ dx$$

$$I = \frac{x^2}{2} arc\ tg\ x - \frac{1}{2}\int \left(1 - \frac{1}{x^2+1}\right) dx$$

$$I = \frac{x^2}{2} arc\ tg\ x - \frac{1}{2} x + \frac{1}{2} arc tg\ x + c$$

$$I = \frac{x^2+1}{2} arc\ tg\ x - \frac{1}{2} x + c$$

29. $\int x^5 e^{x^2}\, dx$

$$u = x^4 \quad \Rightarrow \quad du = 4x^3 dx$$

$$dv = xe^{x^2} dx \Rightarrow v = \int xe^{x^2} dx = \frac{1}{2} e^{x^2}$$

$$I = x^4 \cdot \frac{1}{2} e^{x^2} - \int \frac{1}{2} e^{x^2} \cdot 4x^3 dx$$

$$u = x^2 \quad \Rightarrow \quad du = 2xdx$$

$$dv = xe^{x^2} dx \Rightarrow v = \int xe^{x^2} dx = \frac{1}{2} e^{x^2}$$

$$I = x^4 \cdot \frac{1}{2} e^{x^2} - 2\left[x^2 \cdot \frac{1}{2} e^{x^2} - \int \frac{1}{2} e^{x^2} \cdot 2xdx\right]$$

$$I = x^4 \cdot \frac{1}{2} e^{x^2} - 2x^2 \cdot \frac{1}{2} e^{x^2} + e^{x^2} + c$$

$$I = \frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + c.$$

30. $\int x \cos^2 x\, dx$

$$\int x \frac{1+\cos 2x}{2} dx$$

$$I = \frac{1}{2}\int x\, dx + \frac{1}{2}\int x \cos 2x\, dx$$

$$u = x \quad \Rightarrow \quad du = dx$$

$$dv = \cos 2x \Rightarrow v = \frac{1}{2}\, sen\ 2x$$

$$I = \frac{1}{2}\,\frac{x^2}{2} + \frac{1}{2}\left[x\,\frac{1}{2}\,sen\,2x - \int \frac{1}{2}\,sen\ 2x\ dx\right]$$
$$= \frac{x^2}{4} + \frac{1}{4}\,x\ sen\ 2x - \frac{1}{4}\,.\,\frac{1}{2}\,(-\cos 2x) + c$$
$$= \frac{x^2}{4} + \frac{1}{4}\,x\ sen\ 2x + \frac{1}{8}\cos 2x + c$$

31. $\int (x+3)^2\ e^x\ dx$

$$u = (x+3)^2 \quad \Rightarrow \quad du = 2(x+3)\,dx$$
$$dv = e^x dx \quad \Rightarrow \quad v = e^x$$
$$I = (x+3)^2\ e^x - \int e^x\ 2(x+3)\ dx$$
$$u = x+3 \quad \Rightarrow \quad du = dx$$
$$dv = e^x dx \quad \Rightarrow \quad v = e^x$$
$$I = (x+3)^2\ e^x - 2\left[(x+3)\ e^x - \int e^x\ dx\right]$$
$$= (x+3)^2\ e^x - 2\,(x+3)\ e^x + 2\ e^x + c$$
$$= e^x\left[x^2 + 6x + 9 - 2x - 6 + 2\right] + c$$
$$= e^x\left[x^2 + 4x + 5\right] + c$$

32. $\int x\sqrt{x+1}\ dx$

$$u = x \quad \Rightarrow \quad du = dx$$
$$dv = \sqrt{x+1}dx \Rightarrow v = \frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}}$$
$$I = x\,\frac{2}{3}(x+1)^{\frac{3}{2}} - \int \frac{2}{3}\,(x+1)^{\frac{3}{2}}\ dx$$
$$= \frac{2}{3}\,x(x+1)^{\frac{3}{2}} - \frac{2}{3}\,\frac{(x+1)^{\frac{5}{2}}}{\frac{5}{2}} + c$$
$$= \frac{2}{3}\,x(x+1)^{\frac{3}{2}} - \frac{4}{15}(x+1)^{\frac{5}{2}} + c$$

33. $\int \cos(\ln x)\ dx$

$$u = \cos(\ln x) \quad \Rightarrow \quad du = -sen\,(\ln x)\frac{1}{x}dx$$

$$dv = dx \Rightarrow v = x$$

$$I = \cos(\ln x)x + \int x\,sen(\ln x)\,\frac{dx}{x}$$

$$u = sen(\ln x) \Rightarrow du = \cos(\ln x)\frac{1}{x}dx$$

$$dv = dx \Rightarrow v = x$$

$$I = x\cos(\ln x) + sen(\ln x)x - \int x\cos(\ln x)\,\frac{du}{x}$$

$$I = x\cos(\ln x) + x\,sen(\ln x) - I$$

$$2I = x\cos(\ln x) + x\,sen(\ln x)$$

$$I = \frac{1}{2}(x\cos(\ln x) + x\,sen(\ln x)) + c$$

34. $\int arc\cos x\,dx$

$$u = arc\cos x \quad \Rightarrow \quad du = \frac{-1}{\sqrt{1+x^2}}dx$$

$$dv = dx \Rightarrow v = x$$

$$I = x\,arc\cos x - \int x\,\frac{-dx}{\sqrt{1-x^2}}$$

$$= x\,arc\cos x - \sqrt{1-x^2} + c$$

35. $\int \sec^3 x\,dx$

$$u = \sec x \quad \Rightarrow \quad du = \sec x\,tg\,x\,dx$$

$$dv = \sec^2 x\,dx \quad \Rightarrow \quad v = tg\,x$$

$$I = \sec x.tg\,x - \int tg^2 \sec x\,\,dx$$

$$= \sec x.tg\,x - \int \sec x\left(\sec^2 x - 1\right)dx$$

$$= \sec x.tg\,x - \int \sec^3 x + \int \sec x\,dx$$

$$I = \sec x.tg\,x - I + \ln\left|\sec x + tg\,x\right| + c$$

$$I = \frac{1}{2}\left[\sec x\,.tg\,x + \ln\left|\sec x + tg\,x\right|\right] + c$$

Obs.

$$\int \sec x \, dx = \int \frac{\sec x(\sec x + tg\, x)}{\sec x + tg\, x} \, dx =$$

$$= \int \frac{\sec^2 x + \sec x . tg\, x}{\sec x + tg\, x} dx = \ln \left|\sec x + tg\, x\right| + c,$$

onde utilizamos:

$u = \sec + tg\, x$

$du = \sec . tg\, x + \sec^2 x$

36. $\int \frac{1}{x^3} e^{1/x} dx$

$$u = \frac{1}{x} \Rightarrow du = \frac{-1}{x^2} dx$$

$$dv = \frac{1}{x^2} e^{1/x} dx \Rightarrow v = \int \frac{1}{x^2} e^{1/x} dx = -e^{1/x}$$

$$I = -\frac{1}{x} e^{1/x} - \int - e^{1/x} \frac{-1}{x^2} dx$$

$$= -\frac{1}{x} e^{1/x} + e^{1/x} + c$$